CALCULAREA UNEI SUME DE DOI RADICALI DE ORDIN 3 CONJUGAȚI

Calculați:

$\cancel{1)}$ $\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}$

$\cancel{2)}$ $\sqrt[3]{2+\sqrt{5}}-\sqrt[3]{2-\sqrt{5}}$

3) $\sqrt[3]{5\sqrt{2}+7}-\sqrt[3]{5\sqrt{2}-7}$

4) $\sqrt[3]{10-4\sqrt{6}}+\sqrt[3]{10+4\sqrt{6}}$

5) $\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}$

6) $\sqrt[3]{20+14\sqrt{2}}-\sqrt[3]{20-14\sqrt{2}}$

7) $\sqrt[3]{5\sqrt{2}+7}+\sqrt[3]{5\sqrt{2}-7}$

8) $\sqrt[3]{10+4\sqrt{6}}-\sqrt[3]{10-4\sqrt{6}}$

9) $\sqrt[3]{7-\sqrt{41}}+\sqrt[3]{7+\sqrt{41}}$

10) $\sqrt[3]{8+\sqrt{37}}-\sqrt[3]{8-\sqrt{37}}$

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E_1) se notează cu N numărul cerut $\Rightarrow N=\sqrt[3]{a}\pm\sqrt[3]{b}$

E_2) se calculează N^3 (se ridică la puterea a 3-a)

folosind: $(a+b)^3=a^3+b^3+3ab(a+b)$ sau

$$(a-b)^3=a^3-b^3-3ab(a-b)$$

E_3) se înlocuiește $a+b$ sau $a-b$ cu $N \Rightarrow$ ec. de gr III

E_4) se face Horner $\Rightarrow \begin{cases} N_1=d \\ \text{sau} \\ \text{ec. gr II în } N=0 \text{ care are întotdeauna } \Delta<0 \end{cases}$

Ex.: Calculați: $N=\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}$

E_1) $N=\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}} \,\Big|\, (\ )^3 \Rightarrow N^3=\left(\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}\right)^3 \Rightarrow$

$N^3=20+\cancel{14\sqrt{2}}+20-\cancel{14\sqrt{2}}+3\sqrt[3]{(20+14\sqrt{2})(20-14\sqrt{2})}\left(\sqrt[3]{20+14\sqrt{2}}+\sqrt[3]{20-14\sqrt{2}}\right)$

E_2) $N^3=40+3\sqrt[3]{20^2-(14\sqrt{2})^2}\cdot N \Rightarrow N^3=40+3\sqrt[3]{8}\cdot N \Rightarrow$

$\Rightarrow N^3=40+6N \Rightarrow N^3-6N-40=0$

E_3) Horner, $D_{-40}=\{\pm1;\pm2;\pm4;\pm5;\ldots\}$

	N^3	N^2	N	N^0
	1	0	-6	-40
1	1	1	-5	$\boxed{-45}\neq 0$
2	1	2	-2	$\boxed{-44}\neq 0$
4	1	4	10	$\boxed{0}$

$\Rightarrow (N-4)(N^2+4N+10)=0 \Rightarrow \begin{cases} N=4 \\ \text{sau} \\ N^2+4N+10=0 \end{cases}$

$\Delta=16-40<0$ fals

$\Rightarrow N=4$

Calculați: $\sqrt[3]{2+\sqrt{5}}-\sqrt[3]{2-\sqrt{5}}$

E_1) folosim $(a-b)^3=a^3-b^3-3ab(a-b)$

E_2) notăm $N=\sqrt[3]{2+\sqrt{5}}-\sqrt[3]{2-\sqrt{5}} \quad |(\ )^3$

$\Rightarrow N^3=(2+\sqrt{5})-(2-\sqrt{5})-3\sqrt[3]{(2+\sqrt{5})(2-\sqrt{5})}\cdot N$

$\Rightarrow N^3=\cancel{2}+\sqrt{5}-\cancel{2}+\sqrt{5}-3\sqrt[3]{4-5}\cdot N$

$N^3=2\sqrt{5}-3(-1)N$

$N^3=2\sqrt{5}+3N$

$\Rightarrow N^3-3N-2\sqrt{5}=0$

E_3) $D_{2\sqrt{5}}=\{\pm 1; \pm 2; \pm\sqrt{5}; \pm 2\sqrt{5}\}$

	N^3	N^2	N	N^0
	1	0	-3	$-2\sqrt{5}$
~~-1~~	~~1~~	~~1~~	~~-2~~	~~$-2-2\sqrt{5}\neq 0$~~
$\sqrt{5}$	1	$\sqrt{5}$	2	$\boxed{0}$

$\Rightarrow (N-\sqrt{5})(1\cdot N^2+\sqrt{5}N+2)=0$

$\Rightarrow$ $N-\sqrt{5}=0 \Rightarrow N=\sqrt{5}$

sau

$N^2+\sqrt{5}N+2=0$

$N_{1,2}=\dfrac{-\sqrt{5}\pm\sqrt{5-16}}{2\cdot 1}$, $\Delta<0 \Rightarrow N_{1,2}\notin\mathbb{R}$

$\Rightarrow \boxed{N=\sqrt{5}}$