ECUAȚII TRIGONOMETRICE LINIARE

Rezolvați:

1) $\sqrt{3}\sin x - \cos x = 1$ în $\mathbb{R}$

2) $\cos x + \sin x = -1$ în $[0; 2\pi)$

3) $\sqrt{2}\sin x - \sqrt{2}\cos x = \sqrt{3}$ în $\mathbb{R}$

4) $\sqrt{3}\cos x - \sin x = 0$ în $[-2\pi; 0)$

5) $-\sin x + \cos x = \sqrt{3}$ în $\mathbb{R}$

6) $\sqrt{3}\sin x - \cos x = 1$ în $[2\pi; 3\pi)$

7) $\sqrt{3}\sin x + \cos x = \sqrt{3}$ în $[-4\pi; -2\pi]$

8) $\sqrt{3}\sin x + \cos x = -\sqrt{2}$ în $\mathbb{R}$

9) $-\cos x + \sin x = -\sqrt{2}$ în $\mathbb{R}$

10) $\cos x + \sqrt{3}\sin x = 0$ în $(3\pi; 4\pi)$

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Rezolvați:

$$\sqrt{3}\sin x - \cos x = 1$$

E1) Folosim: $a\sin x + b\cos x = c \mid : \sqrt{a^2+b^2}$

$$\Rightarrow \frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x = \frac{c}{\sqrt{a^2+b^2}}$$

se alege unghiul din tabel pentru care $\frac{a}{\sqrt{a^2+b^2}} = \cos\varphi$ și $\frac{b}{\sqrt{a^2+b^2}} = \sin\varphi$ sau invers

se folosește formula, se ajunge la ec. fundamental.

E2) $\sqrt{3}\cdot\sin x - \cos x = 1 \mid : \sqrt{(\sqrt{3})^2+(1)^2} = 2$

$$\Rightarrow \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{1}{2}$$

$$\frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \text{ și } \frac{1}{2} = \sin\frac{\pi}{6} \Rightarrow$$

E3) $\Rightarrow \cos\frac{\pi}{6}\cdot\sin x - \sin\frac{\pi}{6}\cdot\cos x = \frac{1}{2}$

$$\Rightarrow \sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

E4) $\frac{1}{2} \in [-1,1] \Rightarrow x - \frac{\pi}{6} \in \{\arcsin\frac{1}{2} + 2k\pi \mid k\in\mathbb{Z}\} \cup \{\pi - \arcsin\frac{1}{2} + 2k\pi \mid k\in\mathbb{Z}\}$

$$x - \frac{\pi}{6} \in \{\frac{\pi}{6} + 2k\pi \mid k\in\mathbb{Z}\} \cup \{\pi - \frac{\pi}{6} + 2k\pi \mid k\in\mathbb{Z}\}$$

$$x \in \{\frac{\pi}{6} + \frac{\pi}{6} + 2k\pi \mid k\in\mathbb{Z}\} \cup \{\pi - \frac{\pi}{6} + \frac{\pi}{6} + 2k\pi \mid k\in\mathbb{Z}\}$$

Rezolvați în $[0; 2\pi)$ ecuația

$$\sin x + \cos x = -1$$

E1) $\sin x + \cos x = -1 \mid : \sqrt{1^2+1^2} = \sqrt{2}$

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = -\frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}; \quad \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$

E2) $\sin x \cos\frac{\pi}{4} + \cos x \sin\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$$\sin\left(x + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \Rightarrow$$

E3) $x + \frac{\pi}{4} \in \{\arcsin\left(-\frac{\sqrt{2}}{2}\right) + 2k\pi \mid k \in \mathbb{Z}\} \cup$

$$\cup \{\pi - \arcsin\left(-\frac{\sqrt{2}}{2}\right) + 2k\pi \mid k \in \mathbb{Z}\}$$

$$x + \frac{\pi}{4} \in \{-\frac{\pi}{4} + 2k\pi \mid k \in \mathbb{Z}\} \cup \{\frac{5\pi}{4} + 2k\pi \mid k \in \mathbb{Z}\}$$

$$x \in \{-\frac{2\pi}{4} + 2k\pi \mid k \in \mathbb{Z}\} \cup \{\frac{4\pi}{4} + 2k\pi \mid k \in \mathbb{Z}\}$$

$$x \in \{-\frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z}\} \cup \{\pi + 2k\pi \mid k \in \mathbb{Z}\}$$

E4) x cu sol. din $[0; 2\pi)$

$k = -1 \Rightarrow x \in \{-\frac{\pi}{2} - \pi; \pi - 2\pi\} \Rightarrow x \in \emptyset$

$k = 0 \Rightarrow x \in \{-\frac{\pi}{2}; \pi\} \Rightarrow x = \pi$

$k = 1 \Rightarrow x \in \{-\frac{\pi}{2} + 2\pi; \pi + 2\pi\} \Rightarrow x = \frac{3\pi}{2}$

$k = 2 \Rightarrow x \in \{-\frac{\pi}{2} + 4\pi; \pi + 4\pi\} \Rightarrow x \in \emptyset$

$S = \{\pi; \frac{3\pi}{2}\}$

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Rezolvați în $[-4\pi, -2\pi]$ ec: $\sqrt{3}\sin x + \cos x = \sqrt{3}$

$E_1)\ \sqrt{3}\sin x + \cos x = \sqrt{3} \ \Big|:\sqrt{3+1}=2 \Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{\sqrt{3}}{2}$

$E_2)\ \sin x \cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} \Rightarrow \sin\left(x+\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$E_3)\ x+\frac{\pi}{6} \in \{\arcsin\frac{\sqrt{3}}{2} + 2k\pi \mid k\in\mathbb{Z}\} \cup \{\pi - \arcsin\frac{\sqrt{3}}{2} + 2k\pi \mid k\in\mathbb{Z}\}$

$x+\frac{\pi}{6} \in \{\frac{\pi}{3} + 2k\pi \mid k\in\mathbb{Z}\} \cup \{\pi - \frac{\pi}{3} + 2k\pi \mid k\in\mathbb{Z}\}$

$x \in \{\frac{\pi}{3} - \frac{\pi}{6} + 2k\pi \mid k\in\mathbb{Z}\} \cup \{\pi - \frac{\pi}{3} - \frac{\pi}{6} + 2k\pi \mid k\in\mathbb{Z}\}$

$$\boxed{x \in \{\frac{\pi}{6} + 2k\pi \mid k\in\mathbb{Z}\} \cup \{\frac{\pi}{2} + 2k\pi \mid k\in\mathbb{Z}\}}$$

$E_4)$ dacă x cerea rezolvare în $\mathbb{R}$, soluția era cea de sus
se cere rezolvare în $[-4\pi, -2\pi] \Rightarrow$ dăm valori lui k

$k=0 \Rightarrow x \in \{\frac{\pi}{6}; \frac{\pi}{2}\} \cap [-4\pi, -2\pi] = \emptyset$

$k=-1 \Rightarrow x \in \{\frac{\pi}{6} - \frac{6}{2\pi}; \frac{\pi}{2} - \frac{2}{2\pi}\} = \{-\frac{11\pi}{6}; -\frac{3\pi}{2}\} \cap [-4\pi, -2\pi] = \emptyset$

$k=-2 \Rightarrow x \in \{\frac{\pi}{6} - \frac{6}{4\pi}; \frac{\pi}{2} - \frac{2}{4\pi}\} = \{-\frac{23\pi}{6}; -\frac{7\pi}{2}\} \cap [-4\pi, -2\pi] = \{-\frac{23\pi}{6}; -\frac{7\pi}{2}\}$

$k=-3 \Rightarrow x \in \{\frac{\pi}{6} - 6\pi; \frac{\pi}{2} - 6\pi\} = \{-\frac{35\pi}{6}; -\frac{11\pi}{2}\} \cap [-4\pi, -2\pi] = \emptyset$

$E_5)\ S_f: x \in \{-\frac{23\pi}{6}; -\frac{7\pi}{2}\}$

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