

# CALCULUL DETERMINANTILOR

Calculați  $\det A$  și, acolo unde e cazul, și  $\det B$  pt:

$$1) A = \begin{pmatrix} \sqrt{2} & 1 \\ 4 & \sqrt{8} \end{pmatrix}, B = \begin{pmatrix} i & 1 \\ 3 & 4+i \end{pmatrix}$$

$$2) A = \begin{pmatrix} \sin \frac{\pi}{6} & \sin \pi \\ \cos \frac{\pi}{3} & \cos \frac{\pi}{2} \end{pmatrix}, B = \begin{pmatrix} \sqrt{12} & 5 \\ -1 & \sqrt{3} \end{pmatrix}$$

$$3) A = \begin{pmatrix} 2-i & 2+i \\ 3-i & 3+i \end{pmatrix}, B = \begin{pmatrix} \log_4 2 & \log_9 3 \\ \log_{25} 5 & \log_{16} 2 \end{pmatrix}$$

$$4) A = \begin{pmatrix} \log_2 3 & \log_7 5 \\ \log_5 7 & \log_3 4 \end{pmatrix} \text{ și } B = \begin{pmatrix} C_4^2 & C_3^1 \\ C_3^2 & C_4^3 \end{pmatrix}$$

$$5) A = \begin{pmatrix} 2 & -3 & \sqrt{2} \\ -1 & 0 & 2 \\ 4 & \sqrt{2} & -1 \end{pmatrix},$$

$$B = \begin{pmatrix} \sqrt{3} & \sqrt{2} & \sqrt{6} \\ -\sqrt{6} & \sqrt{3} & \sqrt{2} \\ \sqrt{2} & -\sqrt{6} & -\sqrt{3} \end{pmatrix}$$

$$6) A = \begin{pmatrix} 0! & 1! & 2! \\ P_2 & A_3^2 & -C_3^1 \\ -A_4^3 & C_2^1 & -P_2 \end{pmatrix} \text{ și}$$

$$B = \begin{pmatrix} \log_2 1 & \log_3 3 & \log_4 2 \\ \log_5 5 & -\log_3 9 & \log_2 8 \\ -\log_5 1 & \log_2 4 & -\log_3 27 \end{pmatrix}$$

$$7) A \in M_4(\mathbb{C}) \text{ dată de } a_{ij} = \max\{i, j\}$$

$$8) A \in M_3(\mathbb{N}) \text{ dată de } a_{ij} = \begin{cases} 1, & \text{dacă } i+j \leq 4 \\ 2, & \text{dacă } i+j > 4 \end{cases}$$

Calculati det A, unde  $a_{ij} = \max\{i, j\}$ ,  
 $i, j = \overline{1, 4}$

$$E_1) A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \text{ cu } a_{ij} = \max\{i, j\}$$

$$E_2) a_{11} = \max\{1, 1\} = 1$$

$$a_{12} = \max\{1, 2\} = 2$$

etc

$$a_{44} = \max\{4, 4\} = 4$$

$$E_3) \Rightarrow A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}$$

$$E_4) \det A = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{vmatrix} \xrightarrow{c_2 - c_1 \rightarrow c_2} \begin{vmatrix} 1 & 1 & 3 & 4 \\ 2 & 0 & 3 & 4 \\ 3 & 0 & 3 & 4 \\ 4 & 0 & 4 & 4 \end{vmatrix}$$

$$= (-1)^{1+2} \cdot 1 \cdot \begin{vmatrix} 2 & 3 & 4 \\ 3 & 3 & 4 \\ 4 & 4 & 4 \end{vmatrix} \xrightarrow{\text{factor pe } c_3} -1 \cdot 4 \cdot \begin{vmatrix} 2 & 3 & 4 \\ 3 & 3 & 4 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= -4 \cdot (6 + 12 + 3 - 12 - 8 - 9) = -4 \cdot (-8) = 32$$

Fie  $A = (a_{ij}) \in M_3(\mathbb{N})$ ,

$$a_{ij} = \begin{cases} 1, & \text{dacă } i+j \leq 4 \\ 2, & \text{dacă } i+j > 4 \end{cases}$$

Calculati det A

$$E_1) A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$E_2) \begin{array}{l} a_{11} \\ a_{12} \\ a_{13} \end{array} \begin{array}{l} \underline{\underline{1+1=2 \leq 4}} \\ \underline{\underline{1+2=3 \leq 4}} \\ \underline{\underline{1+3=4 \leq 4}} \end{array} \begin{array}{l} 1 \\ 1 \\ 1 \end{array} \left| \begin{array}{l} a_{21} = 1 \\ a_{22} = 1 \\ a_{23} = 2 \end{array} \right| \begin{array}{l} a_{31} = 1 \\ a_{32} = 2 \\ a_{33} = \mathbf{2} \end{array}$$

$$E_3) A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 2 \end{pmatrix}$$