

CALCUL DETERMINANTILOR PRIN FORMARE DE ZERURI

Prin formare de zeruri,
calculati:

$$1) \Delta = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 2 & 5 \\ 1 & 4 & 2 \end{vmatrix}$$

$$2) \Delta = \begin{vmatrix} 4 & 2 & 1 \\ 3 & -1 & 4 \\ -2 & 3 & 5 \end{vmatrix}$$

$$3) \Delta = \begin{vmatrix} 2 & 3 & 4 & 2 \\ 4 & 1 & 3 & 2 \\ 2 & 2 & 3 & 1 \\ 4 & 2 & 3 & 5 \end{vmatrix}$$

$$4) \Delta = \begin{vmatrix} 2 & 3 & 4 & 5 \\ 2 & 1 & 0 & 3 \\ 4 & 2 & 1 & -1 \\ 3 & 2 & 5 & 2 \end{vmatrix}$$

$$5) \Delta = \begin{vmatrix} 4 & -1 & 2 & -3 \\ -2 & 5 & -4 & 3 \\ 2 & 1 & -2 & 4 \\ 4 & 3 & -2 & 5 \end{vmatrix}$$

$$6) \Delta = \begin{vmatrix} -2 & -1 & 0 & 2 \\ 1 & 3 & 0 & 1 \\ 4 & 2 & 5 & -1 \\ -2 & -3 & 1 & -2 \end{vmatrix}$$

$$7) \Delta = \begin{vmatrix} 4 & 2 & 3 & 1 \\ 3 & 2 & 1 & -4 \\ -1 & -2 & 3 & -2 \\ -4 & -2 & -1 & 1 \end{vmatrix}$$

$$8) \begin{vmatrix} 1 & x & x^2 & x^3 \\ 1 & y & y^2 & y^3 \\ 1 & z & z^2 & z^3 \\ 1 & x+y+z & x^2+y^2+z^2 & x^3+y^3+z^3 \end{vmatrix}$$

$$9) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+a^2 & 1 & 1 \\ 1 & 1 & 1+b^2 & 1 \\ 1 & 1 & 1 & 1+c^2 \end{vmatrix} \text{ unde}$$

a, b, c sunt în orice pozitie
cu rata x

Calculati

$$\Delta = \begin{vmatrix} 2 & 3 & 4 & 2 \\ 4 & 1 & 3 & 2 \\ 2 & 2 & 3 & 1 \\ 4 & 2 & 3 & 5 \end{vmatrix}$$

min formare de zero-uri

$$E_1) \Delta = \begin{vmatrix} 2 & 3 & 4 & 2 \\ 4 & 1 & 3 & 2 \\ 2 & 2 & 3 & 1 \\ 4 & 2 & 3 & 5 \end{vmatrix} \begin{array}{l} C_1 - 4C_2 \rightarrow C_1 \\ C_3 - 3C_2 \rightarrow C_3 \\ C_4 - 2C_2 \rightarrow C_4 \end{array} \begin{vmatrix} -10 & 3 & -5 & -4 \\ 0 & 1 & 0 & 0 \\ -6 & 2 & -3 & -3 \\ -4 & 2 & -3 & 1 \end{vmatrix} =$$

$$E_2) \Delta = (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -10 & -5 & -4 \\ -6 & -3 & -3 \\ -4 & -3 & 1 \end{vmatrix}$$

$$E_3) \Delta = \cancel{30} - 72 - 60 + 48 + 90 - \cancel{30} = \\ = -132 + 138 = 6$$

Calculați: $\begin{vmatrix} 2 & 3 & 4 & 5 \\ 2 & 1 & 0 & 3 \\ 4 & 2 & 1 & -1 \\ 3 & 2 & 5 & 2 \end{vmatrix}$ prin formarea
de zero-uri

E₁) $\Delta \begin{array}{l} C_1 - 2C_2 \rightarrow C_1 \\ C_4 - 3C_2 \rightarrow C_4 \end{array} \begin{vmatrix} -4 & 3 & 4 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & -7 \\ -1 & 2 & 5 & -4 \end{vmatrix} =$

$= (-1)^{2+2} \cdot 1 \cdot \begin{vmatrix} -4 & 4 & -4 \\ 0 & 1 & -7 \\ -1 & 5 & -4 \end{vmatrix} =$

$= (16 + 28 - 0) - (4 + 140 + 0) =$
 $= 16 + 28 - 144 = 44 - 144 = -100$

Calculati

$$\Delta = \begin{vmatrix} 1 & x & x^2 & x^3 \\ 1 & y & y^2 & y^3 \\ 1 & z & z^2 & z^3 \\ 1 & x+y+z & xy+yz+zx & xyz \end{vmatrix}$$

$$\Delta \begin{array}{l} \underline{l_2 - l_1 \rightarrow l_2} \\ \underline{l_3 - l_1 \rightarrow l_3} \\ \underline{l_4 - l_1 \rightarrow l_4} \end{array} \begin{vmatrix} 1 & x & x^2 & x^3 \\ 0 & y-x & y^2-x^2 & y^3-x^3 \\ 0 & z-x & z^2-x^2 & z^3-x^3 \\ 0 & y+z & xy+yz+zx+2x-x^2 & xyz-x^3 \end{vmatrix} =$$

$$= (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} y-x & (y-x)(y+x) & (y-x)(y^2+yx+x^2) \\ z-x & (z-x)(z+x) & (z-x)(z^2+zx+x^2) \\ y+z & xy+yz+zx-x^2 & xyz-x^3 \end{vmatrix}$$

factori

$$\underbrace{(y-x)(z-x)}_{N_1} \begin{vmatrix} 1 & y+x & y^2+yx+x^2 \\ 1 & z+x & z^2+zx+x^2 \\ y+z & xy+yz+zx-x^2 & xyz-x^3 \end{vmatrix}$$

$$\begin{array}{l} \underline{l_2 - l_1 \rightarrow l_2} \\ \underline{l_3 - (y+z) \cdot l_1 \rightarrow l_3} \end{array} N_1 \cdot \begin{vmatrix} 1 & y+x & y^2+yx+x^2 \\ 0 & z-y & z^2-y^2+2x-yx \\ 0 & xy+yz+zx-x^2 - (y+z)(y+x) & xyz-x^3 - (y+z)(y^2+yx+x^2) \end{vmatrix}$$

$$= N_1 \cdot (z-y) \cdot \begin{vmatrix} 1 & z+y+x & z^2+y^2+2xy+2x^2 \\ \cancel{xy+yz+zx-x^2} & \cancel{z-y} & \cancel{z^2-y^2+2x-yx} \\ \cancel{-y^2-yx-zy-zx} & \cancel{-y^2-yx-zy-zx} & \cancel{xyz-x^3 - (y+z)(y^2+yx+x^2)} \end{vmatrix}$$

$$= N_2 \cdot \begin{vmatrix} 1 & z+y+x \\ -x^2-y^2 & -x^3-y^3-y^2x-yx^2-zy^2-zx^2 \end{vmatrix} \xrightarrow{\text{calcul}} N_2 \cdot 0 = 0$$

Dacă a, b, c sunt în progres geometrică cu
rație r , se cere

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1+a^2 & 1 & 1 \\ 1 & 1 & 1+b^2 & 1 \\ 1 & 1 & 1 & 1+c^2 \end{vmatrix}$$

$E_1) \Delta \begin{array}{l} \underline{C_2 - C_1 \rightarrow C_2} \\ C_3 - C_1 \rightarrow C_3 \\ C_4 - C_1 \rightarrow C_4 \end{array} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & a^2 & 0 & 0 \\ 1 & 0 & b^2 & 0 \\ 1 & 0 & 0 & c^2 \end{vmatrix}$

$$= (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{vmatrix} =$$

$$= a^2 b^2 c^2 = (abc)^2$$

$E_2)$ dar, a, b, c p. g. cu rație r
 $\Rightarrow b = a \cdot r, c = a \cdot r^2 \Rightarrow abc = a^3 r^3$

$$\Rightarrow \Delta = (abc)^2 = (a^3 r^3)^2 = a^6 r^6$$