RANGUL UNEI MATRICI

Calculați rang A (sau discutați rang A în funcție de parametrii) pentru:

1) $A=\begin{pmatrix}1&2\\3&4\end{pmatrix}$, $B=\begin{pmatrix}1&2\\2&4\end{pmatrix}$

2) $A=\begin{pmatrix}1&2&3\\2&4&1\\3&6&4\end{pmatrix}$, $B=\begin{pmatrix}1&2&3\\2&4&1\\3&6&-1\end{pmatrix}$

3) $A=\begin{pmatrix}2&2&2\\1&1&1\\3&3&3\end{pmatrix}$, $B=\begin{pmatrix}2&3&1\\1&2&3\\3&5&4\end{pmatrix}$

4) $A=\begin{pmatrix}1&2&-1\\3&6&-3\end{pmatrix}$, $B=\begin{pmatrix}2&-1\\4&-2\\1&3\end{pmatrix}$

5) $A=\begin{pmatrix}2&-1&2&3\\4&-2&4&5\end{pmatrix}$, $B=\begin{pmatrix}1&3&0\\2&6&1\\-1&0&2\\0&1&1\end{pmatrix}$

6) $A=\begin{pmatrix}2&a\\4&6\end{pmatrix}$, $B=\begin{pmatrix}b-1&2\\4&b+1\end{pmatrix}$

7) $A=\begin{pmatrix}1&2&3\\4&a&b\end{pmatrix}$, $B=\begin{pmatrix}2&4&4\\a&1&b\end{pmatrix}$

8) $A=\begin{pmatrix}1&1&2\\2&a&3\\3&-1&2\end{pmatrix}$, $B=\begin{pmatrix}2&1&1\\3&2&1\\4&b&2\end{pmatrix}$

9) $A=\begin{pmatrix}a&1\\1&a\\2&a+1\end{pmatrix}$, $B=\begin{pmatrix}a&-2&1\\-2&1&a\\1&a&-2\end{pmatrix}$

10) $A=\begin{pmatrix}1&1&m\\1&m&m\\m&1&1\end{pmatrix}$, $B=\begin{pmatrix}a&1\\1&a\\2&a+1\end{pmatrix}$

11) $A=\begin{pmatrix}m&1&1&1\\1&m&1&1\\1&1&m&1\\1&1&1&m\end{pmatrix}$

12) găsiți $a, b \in \mathbb{R}$ dacă rang $A=2$ pt $A=\begin{pmatrix}2&3&1&b\\4&a&2&8\end{pmatrix}$

13) $a, b = ?$ dacă rang $A=2$, $A=\begin{pmatrix}-1&1&4\\1&a&-4\\-1&1&b\end{pmatrix}$

14) $\alpha, \beta = ?$ dacă rang $A=2$, $A=\begin{pmatrix}\beta&1&2&4\\1&\alpha&2&3\\1&2\alpha&2&4\end{pmatrix}$

Studiați rang A dacă
$$A=\begin{pmatrix} m & 1 & 1 & 1\\ 1 & m & 1 & 1\\ 1 & 1 & m & 1\\ 1 & 1 & 1 & m\end{pmatrix}$$

E_1) $\det A \xlongequal{l_4+l_3+l_2+l_1\to l_1} \begin{vmatrix} m+3 & m+3 & m+3 & m+3\\ 1 & m & 1 & 1\\ 1 & 1 & m & 1\\ 1 & 1 & 1 & m\end{vmatrix}$

E_2) $\det A \xlongequal{\text{factor pe } l_1} (m+3)\begin{vmatrix} 1 & 1 & 1 & 1\\ 1 & m & 1 & 1\\ 1 & 1 & m & 1\\ 1 & 1 & 1 & m\end{vmatrix}$

$$\xlongequal[\substack{c_3-c_1\to c_3\\ c_4-c_1\to c_4}]{c_2-c_1\to c_2} \begin{vmatrix} 1 & 0 & 0 & 0\\ 1 & m-1 & 0 & 0\\ 1 & 0 & m-1 & 0\\ 1 & 0 & 0 & m-1\end{vmatrix}\cdot(m+3)=$$

$$=(m+3)\cdot(-1)^{1+1}\cdot 1\cdot\begin{vmatrix} m-1 & 0 & 0\\ 0 & m-1 & 0\\ 0 & 0 & m-1\end{vmatrix}$$

$$=(m+3)\left((m-1)^3-0-0-0-0-0\right)=(m+3)(m-1)^3$$

E_3) Caz I: $(m+3)(m-1)^3\neq 0 \iff (m\neq -3$ și $m\neq 1) \Rightarrow \operatorname{rang} A=4$

Caz II: $(m+3)(m-1)^3=0 \Rightarrow m=-3$ sau $m=1$

Caz II.1: $m=-3 \Rightarrow A=\begin{pmatrix} -3 & 1 & 1 & 1\\ 1 & -3 & 1 & 1\\ 1 & 1 & -3 & 1\\ 1 & 1 & 1 & -3\end{pmatrix}$ și $\exists \begin{vmatrix} -3 & 1 & 1\\ 1 & -3 & 1\\ 1 & 1 & -3\end{vmatrix}\neq 0 \Rightarrow \operatorname{rang} A=3$

Caz II.2: $m=1 \Rightarrow A=\begin{pmatrix} 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\\ 1 & 1 & 1 & 1\end{pmatrix} \Rightarrow \operatorname{rang} A=1$

Găsiți $a, b \in \mathbb{R}$ dacă rang $A = 2$

unde $A = \begin{pmatrix} 2 & 3 & 1 & b \\ 4 & a & 2 & 8 \end{pmatrix}$

E1) $\exists$ minorul $|2| = 2 \neq 0 \Rightarrow \text{rang } A \geq 1$

E2) bordăm $\Rightarrow \begin{vmatrix} 2 & 3 \\ 4 & a \end{vmatrix} = 2a - 12$

Caz I: dacă $2a - 12 \neq 0 \Leftrightarrow a \neq 6$

$\Rightarrow \text{rang } A = 2 \quad \forall b \in \mathbb{R}$

Caz II: dacă $2a - 12 = 0 \Leftrightarrow a = 6$

$\Rightarrow A = \begin{pmatrix} 2 & 3 & 1 & b \\ 4 & 6 & 2 & 8 \end{pmatrix}$

$\forall$ minor de ord II ce nu conține variabila b este nul, calc. $\begin{vmatrix} 1 & b \\ 2 & 8 \end{vmatrix} = 8 - 2b$

Caz II 1: $b \neq 4 \Rightarrow \text{rang } A = 2$

Caz II 2: $b = 4 \Rightarrow A = \begin{pmatrix} 2 & 3 & 1 & 4 \\ 4 & 6 & 2 & 8 \end{pmatrix}$

$\Rightarrow \forall$ minor de ord II e nul $\Rightarrow \text{rang } A = 1$ fals

E3) $\text{rang } A = 2 \Leftrightarrow \begin{cases} a \neq 6, \ b \in \mathbb{R} \\ \text{sau} \\ a = 6, \ b \neq 4 \end{cases}$

Găsiți $a, b = ?$ dacă rang $A = 2$ unde

$$A = \begin{pmatrix} -1 & 1 & 4 \\ 1 & a & -4 \\ -1 & 1 & b \end{pmatrix}$$

E1) nu găsim minor de ord II nenul, așadar

pornim cu $\Delta_1 = |-1| = -1 \neq 0 \Rightarrow \text{rang } A \geq 1$

E2) bordăm $\Rightarrow \begin{vmatrix} -1 & 1 \\ 1 & a \end{vmatrix} = -a-1$

Caz I: $-a-1 \neq 0 \Leftrightarrow a \neq -1 \Rightarrow \text{rang } A \geq 2$

Ptca rang $A=2$ trebuie $\begin{vmatrix} -1 & 1 & 4 \\ 1 & a & -4 \\ -1 & 1 & b \end{vmatrix} = 0$

$\Leftrightarrow -ab+4+\cancel{4}+4a-\cancel{4}-b=0$

$-ab+4a-b=0 \Leftrightarrow ab-4a+b=0$

Așadar, $\boxed{a \neq -1 \text{ și } ab-4a+b=0} \Rightarrow \text{rang } A=2$

Caz II: $-a-1=0 \Leftrightarrow a=-1 \Rightarrow A=\begin{pmatrix} -1 & 1 & 4 \\ 1 & -1 & -4 \\ -1 & 1 & b \end{pmatrix}$

toți minorii de ord II ce nu depind de b

sunt nuli, calc. $\begin{vmatrix} 1 & 4 \\ 1 & b \end{vmatrix} = b-4$

Caz II 1: $b \neq 4 \Rightarrow \text{rang } A \geq 2$, ar trebui $\begin{vmatrix} -1 & 1 & 4 \\ 1 & -1 & -4 \\ -1 & 1 & b \end{vmatrix} = 0$

care e adev deoarece $l_2 = -l_1$ (sau calculăm)

Caz II 2: $b=4 \Rightarrow A=\begin{pmatrix} -1 & 1 & 4 \\ 1 & -1 & -4 \\ -1 & 1 & 4 \end{pmatrix}$ care are rang $A=1$ fals

Așadar $\boxed{b \neq 4, \forall a \in \mathbb{R}}$

Găsiți $\alpha, \beta \in \mathbb{R}$ dacă rang $A=2$

unde $A=\begin{pmatrix} \beta & 1 & 2 & 4 \\ 1 & \alpha & 2 & 3 \\ 1 & 2\alpha & 2 & 4 \end{pmatrix}$

E_1) Obs. că $\exists$ minorul $\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 8-6=2\neq 0$

$\Rightarrow$ rang $A \geq 2$

E_2) Pt ca rang $A=2$, ar trebui ca <u>toți</u> minorii de ordin 3 obținuți prin bordare să fie nuli, adică $\Delta_1=\begin{vmatrix} \beta & 2 & 4 \\ 1 & 2 & 3 \\ 1 & 2 & 4 \end{vmatrix}=0$

și $\Delta_2=\begin{vmatrix} 1 & 2 & 4 \\ \alpha & 2 & 3 \\ 2\alpha & 2 & 4 \end{vmatrix}=0$

E_3) din $\Delta_1=0 \Rightarrow 8\beta+6+\cancel{8}-\cancel{8}-6\beta-8=0$

$\Rightarrow 2\beta-2=0 \Rightarrow \beta=1$

din $\Delta_2=0 \Rightarrow 8+12\alpha+\cancel{8\alpha}-16\alpha-6-\cancel{8\alpha}=0$

$\Rightarrow 2-4\alpha=0 \Rightarrow \alpha=\frac{1}{2}$

E_6) așadar, pt $\boxed{\alpha=\frac{1}{2}}$ și $\boxed{\beta=1}$ $\Rightarrow$ rang $A=2$

Discutați rang $A=?$ dacă $A=\begin{pmatrix} a & 1 \\ 1 & a \\ 2 & a+1 \end{pmatrix}$, $B=\begin{pmatrix} a & -2 & 1 \\ -2 & 1 & a \\ 1 & a & -2 \end{pmatrix}$

rang $B=?$

E_1) $A=\begin{pmatrix} a & 1 \\ 1 & a \\ 2 & a+1 \end{pmatrix}$, $\exists |1|=1\neq 0 \Rightarrow$ rang $A\geq 1$, bordăm

$\Rightarrow \begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}=a^2-1$

E_2) Caz I: dacă $a^2-1\neq 0 \Leftrightarrow a\in\mathbb{R}\setminus\{-1;1\} \Rightarrow \begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}\neq 0 \Rightarrow$

$\Rightarrow$ rang $A\geq 2$. Deoarece $A\in\mathcal{M}_{3,2}(\mathbb{C}) \Rightarrow$ rang $A\leq 2$

$\Rightarrow$ rang $A=2$

E_3) Caz II: dacă $a^2-1=0 \Leftrightarrow a\in\{-1;1\}$

Caz II.1: $a=-1 \Rightarrow A=\begin{pmatrix} -1 & 1 \\ 1 & -1 \\ 2 & 0 \end{pmatrix}$, $\exists |1|=1\neq 0$, bordăm

$\Rightarrow \begin{vmatrix} -1 & 1 \\ 1 & -1 \end{vmatrix}=1-1=0$, bordăm altfel $\Rightarrow \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}=0-2=-2\neq 0$

$\Rightarrow$ rang $A\geq 2$, cum $A\in\mathcal{M}_{3,2}(\mathbb{C}) \Rightarrow$ rang $A\leq 2 \Rightarrow$ rang $A=2$

Caz II.2: $a=1 \Rightarrow A=\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 2 & 2 \end{pmatrix}$, $\exists |1|=1\neq 0$, bordăm

$\Rightarrow \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}=1-1=0$, bordăm altfel $\Rightarrow \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix}=2-2=0$, nu $\exists$ altele

obț. prin bordare $\Rightarrow$ rang $A=1$

E_4) $a\in\mathbb{R}\setminus\{-1;1\} \Rightarrow$ rang $A=2$; $a=-1 \Rightarrow$ rang $A=2$, $a=1 \Rightarrow$ rang $A=1$

E_5) $B=\begin{pmatrix} a & -2 & 1 \\ -2 & 1 & a \\ 1 & a & -2 \end{pmatrix}$, $\det B=\begin{vmatrix} a & -2 & 1 \\ -2 & 1 & a \\ 1 & a & -2 \end{vmatrix}=-6a+7-a^3$
$=-a^3-6a+7$

E_6)

	a^3	a^2	a	a^0
	-1	0	-6	7
1	-1	-1	-7	$\boxed{0}$

$\Rightarrow (a-1)(-a^2-a-7)=0$

$\Rightarrow \begin{cases} a=1 \\ a^2+a+7=0 \Rightarrow \Delta=-27<0 \end{cases}$

E_7) Caz I: $a\neq 1 \Rightarrow \det B\neq 0 \Rightarrow \boxed{\text{rang } B=3}$

Caz II: $a=1 \Rightarrow B=\begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$, $\exists |1|=1\neq 0$, $\exists \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix}=-3\neq 0$

bordăm $\Rightarrow \det B=0 \Rightarrow \boxed{\text{rang } B=2}$