

DETERMINAREA LINOR MATRICI CARE COMUTĂ CU ALTE MATRICI

1) Fi $A = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$, $b \neq 0$.

Dacă $X \in M_2(\mathbb{R})$ cu $AX = XA$,

atunci $X = \begin{pmatrix} u & v \\ v & u \end{pmatrix}$

2) Fi $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$.

Dacă $X \in M_3(\mathbb{R})$ cu $AX = XA$,

atunci $X = \begin{pmatrix} a & 0 & 0 \\ b & a & 0 \\ c & b & a \end{pmatrix}$

3) Fi $A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$. Dacă

$X \in M_2(\mathbb{R})$ cu $AX = XA$,

atunci $X = \begin{pmatrix} x & 0 \\ y & x \end{pmatrix}$

4) Fi $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, dacă

$X \in M_2(\mathbb{R})$ cu $AX = XA$,

atunci $X = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$

5) Fi $A = \begin{pmatrix} 2 & 0 \\ 3 & 2 \end{pmatrix}$, dacă

$X \in M_2(\mathbb{R})$ cu $AX = XA$,

atunci $X = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$

6)

Fie $A = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$, $b \neq 0$. Dacă $X \in M_2(\mathbb{R})$ verifică $AX = XA$, atunci $X = \begin{pmatrix} u & v \\ v & u \end{pmatrix}$

E1) $X = \begin{pmatrix} u & v \\ s & t \end{pmatrix}$

E2) $AX = XA \Rightarrow \begin{pmatrix} a & b \\ b & a \end{pmatrix} \begin{pmatrix} u & v \\ s & t \end{pmatrix} = \begin{pmatrix} u & v \\ s & t \end{pmatrix} \begin{pmatrix} a & b \\ b & a \end{pmatrix}$

$\Rightarrow \begin{pmatrix} au + bs & av + bt \\ bu + as & bv + at \end{pmatrix} = \begin{pmatrix} au + bv & bu + av \\ as + bt & bs + at \end{pmatrix}$

E3) Egalăm $\begin{cases} au + bs = au + bv \\ av + bt = bu + av \\ bu + as = as + bt \\ bv + at = bs + at \end{cases}$

$\Rightarrow \begin{cases} bs = bv \mid b \neq 0 \Rightarrow s = v \\ bt = bu \mid b \neq 0 \Rightarrow t = u \\ bu = bt \mid b \neq 0 \Rightarrow u = t \\ bv = bs \mid b \neq 0 \Rightarrow v = s \end{cases}$

E4) $X = \begin{pmatrix} u & v \\ v & u \end{pmatrix}$