MATRICI CU ELEMENT GENERIC

Găsiţi t în funcţie de x şi y dacă $A(x)\cdot A(y)=A(t)$

1) $A(x)=\begin{pmatrix}1 & 0 & x\\ -x & 1 & -\frac{x^2}{2}\\ 0 & 0 & 1\end{pmatrix}$

2) $A(x)=\begin{pmatrix}1-x & 0 & x\\ 0 & 0 & 0\\ x & 0 & 1-x\end{pmatrix}$

3) $A(x)=\begin{pmatrix}1 & \ln x & 0\\ 0 & 1 & 0\\ 0 & 0 & x\end{pmatrix}, x>0$

4) $A(x)=\begin{pmatrix}1 & 0 & x\\ 0 & e^x & 0\\ 0 & 0 & 1\end{pmatrix}$

5) $A(x)=\begin{pmatrix}1 & 0 & -x\\ x & 1 & -\frac{x^2}{2}\\ 0 & 0 & 1\end{pmatrix}$

6) $A(x)=\begin{pmatrix}1+x & 0 & -x\\ 0 & 0 & 0\\ -x & 0 & 1+x\end{pmatrix}$

7) $A(x)=\begin{pmatrix}1 & -\ln x & 0\\ 0 & 1 & 0\\ 0 & 0 & \frac{1}{x}\end{pmatrix}$

8) $A(x)=\begin{pmatrix}1 & 0 & -x\\ 0 & \frac{1}{e^x} & 0\\ 0 & 0 & 1\end{pmatrix}$

9) $A(x)=\begin{pmatrix}1+3x & 2x\\ -6x & 1-4x\end{pmatrix}$

10) Fie $A=\begin{pmatrix}4 & -6\\ 2 & -3\end{pmatrix}$ şi $X(a)=I_2+aA$. Se ştie că $X(a)X(b)=X(a+b+ab)$, se cere $X(1)X(2)\ldots X(2007)=X(2008!-1)$

Fie $A(x)=\begin{pmatrix} 1+3x & 2x \\ -6x & 1-4x \end{pmatrix}$, arătați că

$$A(x)\circ A(y)=A(x+y-xy),\ \forall x,y\in\mathbb{R}$$

$E_1)$ $A(x)\cdot A(y)=\begin{pmatrix} 1+3x & 2x \\ -6x & 1-4x \end{pmatrix}\begin{pmatrix} 1+3y & 2y \\ -6y & 1-4y \end{pmatrix}=$

$$=\begin{pmatrix} (1+3x)(1+3y)-12xy & 2y(1+3x)+2x(1-4y) \\ -6x(1+3y)-6y(1-4x) & -12xy+(1-4x)(1-4y) \end{pmatrix}=$$

$$=\begin{pmatrix} 1+3x+3y-3xy & 2y+6xy+2x-8xy \\ -6x-18xy-6y+24xy & -12xy+1-4x-4y+16xy \end{pmatrix}\Rightarrow$$

$$\boxed{A(x)A(y)=\begin{pmatrix} 1+3x+3y-3xy & 2x+2y-2xy \\ -6x-6y+6xy & 1-4x-4y+4xy \end{pmatrix}}\ (1)$$

$E_2)$ $A(x+y-xy)=\begin{pmatrix} 1+3(x+y-xy) & 2(x+y-xy) \\ -6(x+y-xy) & 1-4(x+y-xy) \end{pmatrix}\Rightarrow$

$$\Rightarrow\boxed{A(x+y)=\begin{pmatrix} 1+3x+3y-3xy & 2x+2y-2xy \\ -6x-6y+6xy & 1-4x-4y+4xy \end{pmatrix}}\ (2)$$

$E_3)$ Din (1) și (2) $\Rightarrow$

$$A(x)A(y)=A(x+y-xy)$$

Fie $A=\begin{pmatrix}4 & -6\\ 2 & -3\end{pmatrix}$ și $X(a)=I_2+aA$. Se știe că

$X(a)X(b)=X(a+b+ab)$, să se arate:

$$X(1)X(2)\ldots X(2007)=X(2008!-1)$$

E1) $\underline{a+b+ab}=a(1+b)+(b+1)-1=(a+1)(b+1)-1$

E2) $X(a)X(b)=X((a+1)(b+1)-1)\Rightarrow X(1)X(2)=X(2\cdot 3-1)$

$\Rightarrow X(1)\cdot X(2)=X(3!-1)$, arătăm că

$$X(1)X(2)\ldots X(n)=X((n+1)!-1),\ \forall n\geq 1$$

E3) I $P(1)$: $X(1)=X(2!-1)\Leftrightarrow X(1)=X(1)$ adev.

II Pp. $P(k)$ adev $\overset{(?)}{\Longrightarrow}$ $P(k+1)$ adev unde

$P(k)$: $X(1)X(2)\ldots X(k)=X((k+1)!-1)$

$P(k+1)$: $X(1)X(2)\ldots X(k+1)=X((k+2)!-1)$

Știm că $P(k)$ adev $\Rightarrow X(1)X(2)\ldots X(k)=X((k+1)!-1)\ \Big|\cdot_{d} X(k+1)$

$\Rightarrow X(1)X(2)\ldots X(k+1)=X((k+1)!-1)\cdot X(k+1)$

$\xRightarrow{X(a)X(b)=X((a+1)(b+1)-1)} X(((k+1)!-1+1)(k+1+1)-1)=$

$=X((k+1)!(k+2)-1)=X((k+2)!-1)$

$\Rightarrow P(k+1)$ adev $\Rightarrow P(n)$ adev, $\forall n\geq 1$

E4) așadar, $X(1)X(2)\ldots X(n)=X((n+1)!-1)$ și

pt $n=2007\Rightarrow X(1)X(2)\ldots X(2007)=X(2008!-1)$