DETERMINAREA PARAMETRILOR CUNOSCÂND ANUMITE LIMITE

Găsiți $a, b, c \in \mathbb{R}$ dacă:

1) $\lim\limits_{n\to\infty}\left(\sqrt{n^2+2n+5}-an+b\right)=5$

2) $\lim\limits_{n\to\infty}\left(an-\sqrt{-2+bn+n^2}\right)=1$

3) $\lim\limits_{n\to\infty}\left(\sqrt{n^4+2n^3}-an^2-bn-c\right)=0$

4) $\lim\limits_{n}\left(\sqrt[3]{n^3+an^2}-\sqrt{n^2-an}\right)=1$

5) $\lim\limits_{n}\dfrac{2^n+3^n+a^n}{3^n+4^n}=0,\ a>0$

6) $\lim\limits_{n} n^a\left(\sqrt{n+\sqrt{n}}-\sqrt{n-\sqrt{n}}\right)\in\mathbb{R}$

7) $\lim\limits_{n} n\left(an+\sqrt{cn^2+bn+2}\right)=1$

8) $\lim\limits_{n\to\infty}\left(\sqrt[3]{1-n^3}-an-b\right)=0$

9) $\lim\limits_{n\to\infty}\left(\dfrac{n+a}{n+2}\right)^n=e$

10) $\lim\limits_{n}\left(a+\dfrac{n+1}{bn^2+n+2}\right)^{n+2}=\dfrac{1}{e},\ b\neq 0$

11) $\lim\limits_{n}\left(\dfrac{n^2+an+2}{n^2+n+1}\right)=e^2$

12) $\lim\limits_{n}\left(\dfrac{n^2+2n}{n^2+2n+1}\right)^{an^2}=e$

13) $\lim\limits_{n\to\infty}\left(\dfrac{n^2+2}{n+1}-an-b\right)=1$

1) dacă $a+b+1=0$, se cere $\lim\limits_{n}\left(a\sqrt{n+1}+b\sqrt{n+2}+\sqrt{n+3}\right)=0$

Calculați:

$$\lim_{n\to\infty}\left(\sqrt{n^2+2n+5}-an+b\right)=5$$

$$E_1)\ l=\lim_{n\to\infty}\left(\frac{\left(\sqrt{n^2+2n+5}-an\right)\left(\sqrt{n^2+2n+5}+an\right)}{\sqrt{n^2+2n+5}+an}+b\right)=5$$

$$l=\lim_{n\to\infty}\left(\frac{(n^2+2n+5)-(a^2n^2)}{\sqrt{n^2+2n+5}+an}+b\right)=5$$

$$=\lim_{n\to\infty}\left(\frac{n^2(1-a^2)+2n+5}{\sqrt{n^2+2n+5}+an}+b\right)=5$$

Pt ca limita să fie finita $\Rightarrow$ același grad

$\Rightarrow 1-a^2=0 \Rightarrow a^2=1 \Rightarrow a=\pm 1$

$E_2)$ $\boxed{a=1}$ înloc $\Rightarrow \lim_{n\to\infty}\left(\frac{2n+5}{\sqrt{n^2+2n+5}+n}+b\right)=5$

$$\Rightarrow \lim_{n\to\infty}\left(\frac{\cancel{n}\left(2+\frac{5}{n}\right)}{\cancel{n}\left(\sqrt{1+\frac{2}{n}+\frac{5}{n^2}}+1\right)}+b\right)=5 \Rightarrow 1+b=5$$

$\Rightarrow \boxed{b=4}$

$\boxed{a=-1} \Rightarrow$ înloc $\Rightarrow \lim_{n\to\infty}\left(\frac{2n+5}{\sqrt{n^2+2n+5}-n}+b\right)=5$

$$\Rightarrow \lim\left(\frac{n\left(2+\frac{5}{n}\right)}{\sqrt{\ \ \ 2\ \ \ 5}-1}+b\right)=5 \Rightarrow \frac{2}{0}+b=5 \text{ fals}$$

Găsiți a, b, c dacă

$$\lim_{n\to\infty}\left(an-\sqrt{-2+bn+cn^2}\right)=1$$

E1) $l=\lim\limits_{n\to\infty}\dfrac{a^2n^2+2-bn-cn^2}{an+\sqrt{-2+bn+cn^2}}=1 \Leftrightarrow$

$$\Leftrightarrow \lim_{n\to\infty}\frac{n^2(a^2-c)-bn+2}{n\left(a+\sqrt{-\frac{2}{n^2}+\frac{b}{n}+c}\right)}=1$$

E2) Dacă $a^2-c\neq 0 \Rightarrow$ gr $P>$ gr $Q \Rightarrow l=\pm\infty$ K.O.

$\Rightarrow$ e obligatoriu $a^2-c=0 \Rightarrow \boxed{c=a^2}$

E3) $l=\lim\limits_{n\to\infty}\dfrac{-bn+2}{n\left(a+\sqrt{-\frac{2}{n^2}+\frac{b}{n}+c}\right)}=1 \Leftrightarrow$

$$\Leftrightarrow \lim_{n\to\infty}\frac{\cancel{n}\left(-b+\frac{2}{n}\right)}{\cancel{n}\left(a+\sqrt{-\frac{2}{n^2}+\frac{b}{n}+a^2}\right)}=1$$

$$l \xrightarrow{\frac{-b}{a+|a|}} 1$$

E4) dacă $a\geq 0 \Rightarrow |a|=a \Rightarrow l=-\dfrac{b}{2a}=1$

$\Rightarrow \boxed{-b=2a}$ și $a\geq 0$

dacă $a<0 \Rightarrow |a|=-a \Rightarrow l \xrightarrow{\frac{-b}{a-a}} \pm\infty$ fals

Găsiți $a, b, c \in \mathbb{R}$ dacă

$$\lim_{n\to\infty}\left(\sqrt{n^4+2n^3}-an^2-bn-c\right)=0$$

E1) $$l=\lim_{n\to\infty}\left(\sqrt{n^4+2n^3}-(an^2+bn)\right)-c=0$$

$$l=\lim_{n\to\infty}\frac{n^4+2n^3-(an^2+bn)^2}{\sqrt{n^4+2n^3}+an^2+bn}-c=0$$

$$l=\lim_{n\to\infty}\frac{n^4+2n^3-a^2n^4-2abn^3-b^2n^2}{n^2\sqrt{1+\frac{2}{n}}+an^2+bn}-c=0$$

$$l=\lim_{n\to\infty}\frac{n^4(1-a^2)+(2-2ab)n^3-b^2n^2}{n^2\left(\sqrt{1+\frac{2}{n}}+a+\frac{b}{n}\right)}-c=0$$

E2) gr numărător $\leq$ gr numitor $\Rightarrow \begin{cases}1-a^2=0 \Rightarrow a=\pm 1\\ 2-2ab=0\end{cases}$

E3) Caz I: $\boxed{a=1} \Rightarrow l=\lim_{n\to\infty}\frac{(2-2b)n^3-b^2n^2}{n^2\left(\sqrt{1+\frac{2}{n}}+1+\frac{b}{n}\right)}-c=0$

$\Rightarrow 2-2b=0 \Rightarrow \boxed{b=1} \Rightarrow l=\lim_{n\to\infty}\frac{-\cancel{n^2}}{\cancel{n^2}\left(\sqrt{1+\frac{2}{n}}+1+\frac{1}{n}\right)}-c=0$

$\Rightarrow \frac{-1}{2}+c=0 \Rightarrow \boxed{c=\frac{1}{2}}$

Caz II: $a=-1 \Rightarrow l=\lim_{n\to\infty}\frac{(2+2b)n^3-b^2n^2}{n^2\left(\sqrt{1+\frac{2}{n}}-1+\frac{b}{n}\right)}-c=0$

$\Rightarrow 2+2b=0 \Rightarrow b=-1 \Rightarrow l=\lim_{n\to\infty}\frac{-\cancel{n^2}}{\cancel{n^2}\left(\sqrt{1+\frac{2}{n}}-1-\frac{1}{n}\right)}-c=0$

$\frac{1}{0}-c \Rightarrow 0$ fals

Găsiți $a \in \mathbb{R}$ dacă

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3+an^2}-\sqrt{n^2-an}\right)=1$$

E1) apare $\sqrt[3]{\ } - \sqrt{\ } \Rightarrow \nexists$ conjugat

E2) $\lim_{n\to\infty}\left(\sqrt[3]{n^3+an^2}-n+n-\sqrt{n^2-an}\right)=1$

$$\Rightarrow \lim_{n\to\infty}\left(\frac{\cancel{n^3}+an^2-\cancel{n^3}}{\sqrt[3]{(n^3+an^2)^2}+\sqrt[3]{n^3+an^2}\cdot n+n^2}+\frac{\cancel{n^2}-\cancel{n^2}+an}{n+\sqrt{n^2-an}}\right)=1$$

$$\Rightarrow \lim_{n\to\infty}\left(\frac{a\cancel{n^2}}{\cancel{n^2}\left(\sqrt[3]{\left(1+\frac{a}{n}\right)^2}+\sqrt[3]{1+\frac{a}{n}}+1\right)}+\frac{a\cancel{n}}{\cancel{n}\left(1+\sqrt{1-\frac{a}{n}}\right)}\right)=1$$

$$\Rightarrow \frac{a}{1+1+1}+\frac{a}{1+1}=1$$

$$\frac{a}{3}+\frac{a}{2}=1 \;\Big|\cdot 6 \Rightarrow 2a+3a=6$$

$$\boxed{a=\frac{6}{5}}$$