LIMITA INVERSEI UNEI FUNCTII

Știind că funcțiile următoare sunt bijective, se cere:

1) $\lim\limits_{x\to\infty} \dfrac{f^{-1}(x)}{\sqrt[3]{x}}$ pt $f(x)=x^3+x$

2) $\lim\limits_{x\to\infty} \dfrac{f^{-1}(x)}{\sqrt[3]{x}}$ pt $f(x)=-x^3-x+1$

3) $\lim\limits_{x\to 2} \dfrac{f^{-1}(x)}{\sqrt[3]{x}}$ pt $f(x)=x^3+x$

4) $\lim\limits_{x\to 5} \dfrac{f^{-1}(x)}{\ln f(x)}$ pt $f(x)=2^x+3^x$

5) $\lim\limits_{x\to 3} \dfrac{\sqrt[3]{x}}{f^{-1}(x)}$ pt $f(x)=x^3+2x$

6) $\lim\limits_{x\to 1} \dfrac{f^{-1}(x)}{\sqrt[3]{x}}$ pt $f(x)=x^3+1$

7) $\lim\limits_{x\to -\infty} \dfrac{f^{-1}(x)}{\sqrt[3]{x}}$ pt $f(x)=-x^3-x-2$

8) $\lim\limits_{x\to\infty} \dfrac{f^{-1}(x)}{\sqrt[3]{x}}$ pt $f(x)=-x^3-3x+1$

9) $\lim\limits_{x\to 4} \dfrac{\sqrt[3]{x}}{f^{-1}(x)}$ pt $f(x)=x^3+2x+1$

10) $\lim\limits_{x\to -\infty} \dfrac{\sqrt[3]{x}}{f^{-1}(x)}$ pt $f(x)=-x^3+2$

Prof. Ovidiu Bădescu

Știind că f bijectivă, calculați

$$\lim_{x\to\infty} \frac{f^{-1}(x)}{\sqrt[3]{x}} \text{ at } f(x) = -x^3 - x + 1$$

E1) $l = \lim_{x\to\infty} \frac{f^{-1}(x)}{\sqrt[3]{x}}$

E2) not $t = f^{-1}(x) \Leftrightarrow x = f(t)$

$x \to \infty \Rightarrow f(t) \to \infty$

Obs. că $\lim_{t\to-\infty} f(t) = \lim_{t\to-\infty} (-t^3 - t + 1) =$

$= \lim_{t\to-\infty} -t^3\left(1 + \frac{1}{t^2} - \frac{1}{t^3}\right) = \infty$

$\Rightarrow \left.\begin{array}{l}\lim_{t\to-\infty} f(t) = \infty \\ f \text{ bijectivă}\end{array}\right\} \Leftrightarrow f(t) \to \infty \Leftrightarrow t \to -\infty$

E3) $l = \lim_{t\to-\infty} \frac{t}{\sqrt[3]{f(t)}} = \lim_{t\to-\infty} \frac{t}{\sqrt[3]{-t^3 - t + 1}} \overset{\frac{-\infty}{\infty}}{=\!=}$

$= \lim_{t\to-\infty} \frac{\cancel{t}}{\cancel{t}\sqrt[3]{-1 - \frac{1}{t^2} + \frac{1}{t^3}}} = -1$

Prof. Ovidiu Bădescu