SEMNUL FUNCȚIILOR CONTINUE

Stabiliți semnul pentru:

1) $(2^x-2)(x^2-x)(2-x)$

2) $(\ln x-1)(x^2+x)(3^x-9)$

3) $(4^x-2)(3^x-1)(\lg x-1)$

4) $\dfrac{(2^x-1)(9^x-3)}{\ln x-1}$

5) $\dfrac{(\lg^2 x-\lg x-2)(4^x-1)}{(2-4^x)(3-x)}$

6) $\dfrac{(x^2-4)(e^x-1)}{(\ln x-1)(3-x)}$

7) $\dfrac{(x-1)(2^x-4)}{2x-x^2}$

8) $\dfrac{(x^2+x-2)(3^x-9)}{1-\ln x}$

9) $\dfrac{(5^x-1)(3-9^x)}{(\lg x-1)(1-x)}$

10) $f:\Delta\to\mathbb{R},\ f(x)=\begin{cases}\dfrac{5^x-1}{2^x-2}, & x\le 0\\[2mm] \dfrac{\lg x-1}{2^x+2}, & x>0\end{cases}$

11) $f:\Delta\to\mathbb{R},\ f(x)=\begin{cases}\dfrac{\ln x-1}{3^x-3}, & x<2\\[2mm] \dfrac{2^x+3}{4^x-1}, & x\ge 2\end{cases}$

Semnul pentru: $(2^x-2)(x^2-x)(2-x)$

E1) $\nexists$ condiții $\Rightarrow$ rezolvăm pe $\mathbb{R}$

E2) $2^x-2=0 \Rightarrow 2^x=2$, $f(x)=2^x$ injectivă

$\Rightarrow x=1$

$x^2-x=0 \Rightarrow x(x-1)=0 \Rightarrow x_1=0,\ x_2=1$

$2-x=0 \Rightarrow -x=-2 \Rightarrow x=2$

E3)

x		0		1		2	
2^x-2	$-\ -\ -$	$-$	$-\ -$	0	$+$	$+$	$+\ +$
x^2-x	$+\ +\ +$	0	$-\ -\ -$	0	$+$	$+$	$+\ +$
$2-x$	$+\ +\ +$	$+$	$+\ +$	$+$	$+$	0	$-\ -\ -$
$P(x)$	$-\ -\ -$	0	$+\ +$	0	$+$	0	$-\ -\ -$

pt a găsi semnul de la $f(x)=2^x-2$, dăm o valoare , $f(0)=2^0-2=-1<0 \Rightarrow f(x)<0$ pe $(-\infty;1)$

$f(2)=2^2-2=2>0 \Rightarrow f(x)>0$ pe $(1;\infty)$

E4) pt ca $P(x)>0 \Leftrightarrow x\in(0;1)\cup(1;2)$

$P(x)<0 \Leftrightarrow x\in(-\infty;0)\cup(2;\infty)$

$P(x)\geq 0 \Leftrightarrow x\in[0;1]\cup[1;2]=[0;2]$

$P(x)\leq 0 \Leftrightarrow x\in(-\infty;0]\cup[2;\infty)$

Rezolvați: $\dfrac{(\lg^2 x-\lg x-2)(4^x-1)}{(2-4^x)(3-x)}>0$

E_1) cond: $x>0 \Rightarrow$ rezolv doar pe $(0;\infty)$

E_2) $\lg^2 x-\lg x-2=0$, $\lg x=t \Rightarrow t^2-t-2=0$

$\Rightarrow t_{1,2}=\dfrac{1\pm\sqrt{1+8}}{2\cdot 1}=\dfrac{1\pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$

$t_1=2 \Rightarrow \lg x=2 \Rightarrow x=10^2=100$

$t_2=-1 \Rightarrow \lg x=-1 \Rightarrow x=10^{-1}=\dfrac{1}{100}$

$4^x-1=0 \Rightarrow 4^x=1 \Rightarrow 4^x=4^0$, $f(x)=4^x$ inj

$\Rightarrow x=0$

$2-4^x=0 \Rightarrow 4^x=2 \Rightarrow 2^{2x}=2^1$, $f(x)=2^{2x}$ inj

$\Rightarrow 2x=1 \Rightarrow x=\dfrac{1}{2}$

$3-x=0 \Rightarrow x=3$

E_3)

x	0 $\quad\frac{1}{100}\quad\frac{1}{2}\quad 3\quad 100$
$\lg^2 x-\lg x-2$	$+\,+\,+\;0\;-\,-\,-\,-\,-\;0\;+\,+\,+$
4^x-1	$0\;+\,+\,+\,+\,+\,+\,+$
$2-4^x$	$+\,+\,+\,+\;0\;-\,-\,-\,-\,-$
$3-x$	$+\,+\,+\,+\,+\,+\;0\;-\,-\,-$
$F(x)$	$+\,+\,+\,+\;0\;-\,-\,\|\,+\,+\,\|\,-\,-\,-\;0\;+\,+\,+$

pt semnul lui $\lg^2 x-\lg x-2$, calc în $x=\frac{1}{1000}$, 1, 1000

E_4) $F(x)>0 \Leftrightarrow x\in\left(0;\frac{1}{100}\right)\cup\left(\frac{1}{2};3\right)\cup(100;\infty)$, analog pt $F(x)\geq 0$, $F(x)\leq 0$

Stabiliți semnul pentru

$$f(x)=\begin{cases}\dfrac{5^x-1}{2^x-2}, & x\le 0\\[2mm] \dfrac{\lg x-1}{2^x+2}, & x>0\end{cases}$$

$E_1)$ $x\le 0 \Rightarrow f(x)=\dfrac{5^x-1}{2^x-2}$, $5^x-1=0\Rightarrow 5^x=1\Rightarrow$
$5^x=5^0$, $f(x)=5^x$ inj
$\Rightarrow x=0$

$2^x-2=0\Rightarrow 2^x=2$, $f(x)=2^x$ inj $\Rightarrow x=1\notin(-\infty;0]$

x		$0]$	1
5^x-1	$- - - - - - - - -$	0	
2^x-2	$- - - - - - -$		0
$\dfrac{5^x-1}{2^x-2}$	$+ + + +$	$0]$	

$5^{-1}-1=\frac{1}{5}-1<0$

$2^{-1}-2=\frac{1}{2}-2<0$

$E_2)$ $x>0\Rightarrow f(x)=\dfrac{\lg x-1}{2^x+2}$

$\lg x-1=0\Rightarrow \lg x=1\Rightarrow x=10^1\Rightarrow x=10\in(0;\infty)$

$2^x+2=0\Rightarrow 2^x=-2\Rightarrow x\in\emptyset$ pt ca pt $b>0\Rightarrow b^{f(x)}>0$

	$(0$		10	
$\lg x-1$		$- - - - -$	0	$+ + + +$
2^x+2		$+ + + +$	$+$	$+ + + +$
$\dfrac{\lg x-1}{2^x+2}$		$- - - - -$	0	$+ + +$

$x=1\Rightarrow \lg 1-1=-1<0$; $x=100\Rightarrow \lg 100-1=2-1=1>0$

pt 2^x+2, $x=1\Rightarrow 2+2=4>0$

$E_3]$ reunind tabelele $\Rightarrow$

x		0		10	
$f(x)$	$+ + + +$	$0]($	$- - -$	0	$+ + +$

$f(x)<0 \Leftrightarrow x\in(0;10)$

$f(x)>0 \Leftrightarrow x\in(-\infty;0)\cup(10;\infty)$

$f(x)\le 0 \Leftrightarrow x\in[0;10]$

$f(x)\ge 0 \Leftrightarrow x\in(-\infty;0]\cup[10;\infty)$

Stabiliți semnul pentru $f(x) = \begin{cases} \dfrac{\ln x - 1}{3^x - 3}, & x < 2 \\ \dfrac{2^x + 3}{4^x - 1}, & x \geq 2 \end{cases}$

<u>E_1) cond:</u>

E_1) $x \in (-\infty; 2) \Rightarrow f(x) = \dfrac{\ln x - 1}{3^x - 3}$, cond $x > 0 \Rightarrow$ studiem pt $x \in (0; 2)$

$\ln x = 1 \Rightarrow x = e \notin (0; 2)$; $3^x = 3$, $f(x) = 3^x$ inj $\Rightarrow x = 1 \in (0; 2)$

E_2)

x	$(0$		1		$2)$	e
$\ln x - 1$	$-$	$-$	$-$	$-$	$-$	0
$3^x - 3$	$-$	$-$	0	$+$	$+$	
$\dfrac{\ln x - 1}{3^x - 3}$	$($ $+$	$+$	$\vert$	$-$	$-$ $)$	

$x = 1 \Rightarrow \ln 1 - 1 = 0 - 1 < 0$

$x = \frac{1}{2} \Rightarrow 3^{\frac{1}{2}} - 3 = \sqrt{3} - 3 < 0$

$x = \frac{3}{2} \Rightarrow 3^{\frac{3}{2}} - 3 = \sqrt{27} - 3 > 0$

E_3) $x \in [2; \infty) \Rightarrow f(x) = \dfrac{2^x + 3}{4^x - 1}$, nu avem condiții

$2^x + 3 = 0 \Rightarrow 2^x = -3 \Rightarrow x \in \emptyset$

$4^x - 1 = 0 \Rightarrow 4^x = 1 \Rightarrow 4^x = 4^0$, $f(x) = 4^x$ inj $\Rightarrow x = 0 \notin [2; \infty)$

x	$[2$					
$2^x + 3$	$+$	$+$	$+$	$+$	$+$	$+$
$4^x - 1$	$+$	$+$	$+$	$+$	$+$	
$\dfrac{2^x + 3}{4^x - 1}$	$+$	$+$	$+$	$+$	$+$	$+$

$2^3 + 3 = 8 + 3 = 11 > 0$

$4^3 - 1 = 64 - 1 = 63 > 0$

E_4) tabelul final

x	$(0$		1		2				∞
$f(x)$	$($ $+$	$+$	$\vert$	$-$	$-$ $)[$ $+$	$+$	$+$	$+$	

$\Rightarrow f(x) < 0 \Leftrightarrow x \in (1; 2)$

$f(x) > 0 \Leftrightarrow x \in (0; 1) \cup [2; \infty)$

$f(x) \leq 0 \Leftrightarrow x \in (1; 2)$

$f(x) \geq 0 \Leftrightarrow x \in (0; 1) \cup [2; \infty)$