NR. DE RĂDĂCINI REALE ALE UNEI EC. FOLOSIND ŞIRUL LUI ROLLE

Găsiţi numărul de rădăcini reale pentru:

1) $3x^4-8x^3-6x^2+24x-10=0$

2)

3) $2x^3-9x^2-60x+11=0$

4) $x^3+8x\ln x-11x+10=0$

5) . . . .

6) $x^3-6\,\mathrm{arctg}\,x+2=0$

7) $2x-2\,\mathrm{arctg}\,x+\ln(x^2+1)-3=0$

8) $e^x=x+2$

9) $x=\ln x+3$

10) $x+\ln^2 x=1$

11) $x^4+2x^3+3x^2-4=0$

Găsiți numărul de rădăcini reale pentru

$$3x^4-8x^3-6x^2+24x-10=0$$

E1) $f(x)=3x^4-8x^3-6x^2+24x-10$

$f:\mathbb{R}\to\mathbb{R}$

E2) $f'(x)=12x^3-24x^2-12x+24$

$=12(x^3-2x^2-x+2)=0\Rightarrow$

$f'(x)=0\Rightarrow$ $D_2=\{\pm1;\pm2\}$

$x^3-2x^2-x+2=0$

E3)

	x^3	x^2	x	x^0
	1	-2	-1	2
1	1	-1	-2	$\boxed{0}$

$\Rightarrow(x-1)(x^2-x-2)=0\Rightarrow$ $x-1=0\Rightarrow x=1$ sau $x^2-x-2=0$

$\Rightarrow x_{1,2}=\frac{1\pm\sqrt{9}}{2}<\begin{matrix}2\\-1\end{matrix}\Rightarrow x\in\{-1;1;2\}$

E4)

x	$-\infty$	-1	1	2	∞
f'		0	0	0	
f	∞	-29	3	-2	∞

$\lim\limits_{x\to-\infty}f(x)=\lim\limits_{x\to-\infty}(3x^4)=\infty$; $f(-1)=3+8-6-24-10=-29$

$f(1)=3$; $f(2)=-2$; $\lim\limits_{x\to\infty}f(x)=\infty$

E5) șirul Rolle: $(+;-;+;-;+)\Rightarrow$

$x_1\in(-\infty;-1)$, $x_2\in(-1;1)$, $x_3\in(1;2)$, $x_4\in(2;\infty)$

Găsiți numărul de răd. reale pt.:

$$2x^3-9x^2-60x+11=0$$

$E_1)$ nu avem condiții $\Rightarrow f:\mathbb{R}\to\mathbb{R}$

$E_2)$ $f(x)=2x^3-9x^2-60x+11$

$f'(x)=6x^2-18x-60$

$f'(x)=0 \Rightarrow 6(x^2-3x-10)=0$

$\Rightarrow x^2-3x-10=0 \Rightarrow x_{1,2}=\dfrac{3\pm\sqrt{9+40}}{2\cdot 1}=\dfrac{3\pm 7}{2}$ $\nearrow 5$, $\searrow -2$

$E_3)$

x	$-\infty$	-2	5	∞
f'		0	0	
f	$-\infty$	79	-264	∞

$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}(2x^3-9x^2-60x+11)\xrightarrow{\text{factor}}-\infty$

$f(-2)=-16-36+120+11=131-52=$
$=79$

$f(5)=250-225-300+11=-275+11=-264$

$\lim\limits_{x\to\infty} f(x)\xrightarrow{\text{factor}}\infty$

$E_4)$ șirul Rolle: $(-;+;-;+)$

$\Rightarrow x_1\in(-\infty;-2)$, $x_2\in(-2;5)$, $x_3\in(5;\infty)$

Discutați nr. răd. pt ec.:

$$x^3+8x\ln x-11x+10=0$$

E_1) cond.: $x>0 \Rightarrow f:(0;\infty)\to\mathbb{R}$,

$$f(x)=x^3+8x\ln x-11x+10$$

E_2) $f'(x)=3x^2+8\left(\ln x+x\cdot\frac{1}{x}\right)-11$

$$f'(x)=3x^2+8\ln x-3$$

nu pot găsi răd pt $f'(x) \Rightarrow$ calc.

$f''(x)=6x+8\cdot\frac{1}{x}$; $f''(x)=0 \Rightarrow \frac{6x^2+8}{x}=0$

$\Rightarrow 6x^2+8=0$ fals

E_3)

x	0		1		∞
f''		$+$	$+$	$+\ +\ +\ +$	
f'	$-\infty$	$\nearrow$	0	$\nearrow$	∞
f	10		0		∞

$$\lim_{\substack{x\to 0\\x>0}} f'(x)=\lim_{\substack{x\to 0\\x>0}}\left(3x^2+8\ln x-3\right)=-\infty$$

$$\lim_{x\to\infty} f'(x)=\lim_{x\to\infty}\left(3x^2+8\ln x-3\right)\overset{\infty+\infty}{=\!=}\infty$$

$\Rightarrow \exists$ o răd unică pt $f'(x)$ pe $(0;\infty)$

$\Rightarrow$ pot calcula șirul lui Rolle

E_4) obs. că $f'(1)=0$

E_5) $\lim\limits_{\substack{x\to 0\\x>0}} f(x)=\lim\limits_{\substack{x\to 0\\x>0}}\left(x^3+8x\ln x-11x+10\right)$, $l_1=\lim\limits_{\substack{x\to 0\\x>0}} x\ln x \overset{0\cdot(-\infty)}{=\!=}$

$\Rightarrow l_1=\lim\limits_{\substack{x\to 0\\x>0}}\dfrac{\ln x}{\frac{1}{x}}\overset{\frac{-\infty}{\infty}}{\underset{l'H}{=\!=}}\lim\limits_{\substack{x\to 0\\x>0}}\dfrac{\frac{1}{x}}{-\frac{1}{x^2}}=\lim\limits_{\substack{x\to 0\\x>0}}(-x)=0 \Rightarrow l=10$

$f(1)=0$; $\lim f(x)=\infty \Rightarrow$ șirul ROLLE $(+;0;+) \Rightarrow x_1=1$ răd nu e multiplă

Stabiliţi numărul de rădăcini reale ale ecuaţiei $x^4+2x^3+3x^2-4=0$

$E_1)$ $f(x)=x^4+2x^3+3x^2-4$, $f:\mathbb{R}\to\mathbb{R}$

$E_2)$ $f'(x)=4x^3+6x^2+6x$

$f'(x)=0 \Rightarrow 4x^3+6x^2+6x=0 \,/:2$

$2x^3+3x^2+3x=0 \Rightarrow x(2x^2+3x+3)=0$

$\Rightarrow \begin{cases} x_1=0 \\ \text{sau} \\ 2x^2+3x+3=0 \end{cases}$

$\Delta=9-24<0 \Rightarrow \nexists$ răd reale

$E_3)$

x	$-\infty$	0	∞
f'		0	
f	$+\infty$	-4	∞

$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}(x^4+2x^3+3x^2-4)=\lim\limits_{x\to-\infty} x^4=+\infty$

$\lim\limits_{x\to\infty} f(x)=\lim\limits_{x\to\infty}(x^4+2x^3+3x^2-4)=\lim\limits_{x\to\infty} x^4=\infty$

$f(0)=0+0+0-4=-4$

$E_4)$ Şirul lui Rolle: $(+;-;+) \Rightarrow$

$\Rightarrow \exists$ 2 răd reale, $x_1\in(-\infty;0)$ şi $x_2\in(0;\infty)$