REGULILE LUI L'HOPITAL

Calculați:

1) $\lim\limits_{x\to\infty} \dfrac{e^{2x}}{x}$ și $\lim\limits_{x\to\infty} \dfrac{\ln x}{2x+1}$

2) $\lim\limits_{x\to\infty} \dfrac{x^2+2x+3}{\ln(x+2)}$ și $\lim\limits_{\substack{x\to 0\\ x>0}} x^x$

3) $\lim\limits_{x\to\infty} \dfrac{x^2-x+\ln x}{x+2\ln x}$ și $\lim\limits_{x\to 0} (\cos x)^{\frac{1}{x^2+x}}$

4) $\lim\limits_{x\to\infty} \dfrac{\ln(x+e^x)}{\ln(x^2+e^{2x})}$ și $\lim\limits_{x\to 0} \left(\dfrac{1}{x^2}\right)^{\sin x}$

5) $\lim\limits_{x\to\infty} \dfrac{x\cdot e^x}{x^2+1}$ și $\lim\limits_{x\to\infty} x^{\frac{1}{x}}$

6) $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-1}{\sin x}$ și $\lim\limits_{\substack{x\to 0\\ x>0}} x^{\sin x}$

7) $\lim\limits_{x\to 0} \dfrac{\cos 2x-\cos x}{x\sin x}$ și $\lim\limits_{x\to 0} \left(\dfrac{\operatorname{tg} x}{x}\right)^{\frac{1}{x^2}}$

8) $\lim\limits_{x\to 0} \dfrac{2\sin x}{1+x-\cos x}$ și $\lim\limits_{x\to 1} \dfrac{x^5-1}{x^6+2x^3-3}$

9) $\lim\limits_{x\to 2} \dfrac{x^3-8}{x^6-64}$ și $\lim\limits_{x\to 0} \dfrac{1-\cos 3x}{x^2+x}$

10) $\lim\limits_{\substack{x\to 0\\ x>0}} x\ln x$ și $\lim\limits_{x\to 1} \dfrac{\sin(2^x-2)}{\operatorname{tg}(3^x-3)}$

11) $\lim\limits_{x\to -2} \left(\dfrac{1}{x+2}-\dfrac{1}{\ln(x+3)}\right)$ și $\lim\limits_{x\to\infty} \left(\dfrac{\pi}{2}-\operatorname{arctg} x\right)^{\frac{1}{\ln x}}$

Calculați: a) $\lim\limits_{x\to 0} \dfrac{\cos 2x - \cos x}{x \sin x}$

b) $\lim\limits_{x\to 0} \left(\dfrac{tg\, x}{x}\right)^{\frac{1}{x^2}}$

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a) $E_1)$ $l = \lim\limits_{x\to 0} \dfrac{\cos 2x - \cos x}{x \sin x} = \lim\limits_{x\to 0} \left(\dfrac{\cos 2x - \cos x}{x^2} \cdot \dfrac{x}{\sin x}\right)$

$E_2)$ $l_1 = \lim\limits_{x\to 0} \dfrac{\cos 2x - \cos x}{x^2} \underset{l'H}{\overset{\frac{0}{0}}{=\!=}} \lim\limits_{x\to 0} \dfrac{-\sin 2x \cdot 2 + \sin x}{2x} \underset{l'H}{\overset{\frac{0}{0}}{=\!=}}$

$= \lim\limits_{x\to 0} \dfrac{-2\cos 2x \cdot 2 + \cos x}{2} \overset{\frac{-4+1}{2}}{=\!=} -\dfrac{3}{2}$

$E_3)$ $l_2 = \lim\limits_{x\to 0} \dfrac{x}{\sin x} = \lim\limits_{x\to 0} \dfrac{1}{\frac{\sin x}{x}} \overset{\frac{1}{1}}{=\!=} 1$ (limita remarcabila)

$E_4)$ $l = l_1 \cdot l_2 = -\dfrac{3}{2} \cdot 1 = -\dfrac{3}{2}$

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b) $E_1)$ $l = \lim\limits_{x\to 0} \left(\dfrac{tg\, x}{x}\right)^{\frac{1}{x^2}} \overset{1^{\infty}}{=\!=} \lim\limits_{x\to 0} \left(1 + \dfrac{tg\, x}{x} - 1\right)^{\frac{1}{x^2}} =$

$= \lim\limits_{x\to 0} \left[\left(1 + \dfrac{tg\, x - x}{x}\right)^{\frac{x}{tg\, x - x}}\right]^{\frac{tg\, x - x}{x^3}} = e^{\lim\limits_{x\to 0} \frac{tg\, x - x}{x^3}}$

$E_2)$ $l_1 = \lim\limits_{x\to 0} \dfrac{tg\, x - x}{x^3} \underset{l'H}{\overset{\frac{0}{0}}{=\!=}} \lim\limits_{x\to 0} \dfrac{\frac{1}{\cos^2 x} - 1}{3x^2} =$

$= \lim\limits_{x\to 0} \dfrac{1 - \cos^2 x}{3x^2 \cos^2 x} \overset{\frac{0}{0}}{=\!=} \dfrac{1}{3\cos^2 x} \cdot \lim\limits_{x\to 0} \dfrac{1 - \cos^2 x}{x^2}$

$E_3)$ $l_2 = \lim\limits_{x\to 0} \dfrac{1 - \cos^2 x}{x^2} \underset{l'H}{\overset{\frac{0}{0}}{=\!=}} \lim\limits_{x\to 0} \dfrac{-2\cos x\left(-\overset{1}{\cancel{\sin x}}\right)}{2\cancel{x}} = 1$

$E_4)$ $l_1 = \dfrac{1}{3} \Rightarrow l = e^{\frac{1}{3}}$

Calculați: a) $\lim\limits_{\substack{x\to 0\\ x>0}} x\ln x$

b) $\lim\limits_{x\to 1} \dfrac{\sin(2^x-2)}{tg(3^x-3)}$

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a) $E_1)\ l=\lim\limits_{\substack{x\to 0\\ x>0}} x\ln x \overset{0\cdot(-\infty)}{=\!=} \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{\ln x}{\frac{1}{x}} \overset{\frac{-\infty}{\infty}}{\underset{l'H}{=\!=}}$

$=\lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{\substack{x\to 0\\ x>0}} \dfrac{1}{\cancel{x}}\cdot(-x^{\cancel{2}}) = 0$

b) $l=\lim\limits_{x\to 1} \dfrac{\sin(2^x-2)}{tg(3^x-3)} \overset{\frac{0}{0}}{\underset{l'H}{=\!=}} \lim\limits_{x\to 1} \cos(3^x$

$=\lim\limits_{x\to 1} \dfrac{\cos(2^x-2)\cdot(2^x-2)'}{\frac{1}{\cos^2(3^x-3)}\cdot(3^x-3)'} = \lim\limits_{x\to 1} \dfrac{\cos(2^x-2)\cdot 2^x\ln 2}{\frac{1}{\cos^2(3^x-3)}\cdot 3^x\ln 3} =$

$\overset{\frac{1\cdot 2\ln 2}{1\cdot 3\ln 3}}{=\!=} \dfrac{2\ln 2}{3\ln 3}$

Calculați: a) $\lim\limits_{x\to -2}\left(\frac{1}{x+2}-\frac{1}{\ln(x+3)}\right)$

b) $\lim\limits_{x\to\infty}\left(\frac{\pi}{2}-\operatorname{arctg} x\right)^{\frac{1}{\ln x}}$

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a) E_1) $$l=\lim_{x\to -2}\frac{\ln(x+3)-(x+2)}{(x+2)\cdot\ln(x+3)} \overset{\frac{0-0}{0\cdot 0}}{\underset{l'H}{=}}$$

$$=\lim_{x\to -2}\frac{\frac{1}{x+3}\cdot 1-{}^{x+3)}1}{1\cdot\ln(x+3)+(x+2)\cdot\frac{1}{x+3}} \overset{\frac{0}{0}}{=} \lim_{x\to -2}\frac{\frac{1-x-3}{\cancel{(x+3)}}}{\frac{(x+3)\ln(x+3)+x+2}{\cancel{x+3}}}$$

$$=\lim_{x\to -2}\frac{-x-2}{(x+3)\ln(x+3)+x+2} \overset{\frac{0}{0}}{\underset{l'H}{=}} \lim_{x\to -2}\frac{-1}{1\ln(x+3)+\cancel{(x+3)}\frac{1}{\cancel{x+3}}+1}$$

$$=\lim_{x\to -2}\frac{-1}{\ln(x+3)+2} \overset{\frac{-1}{0+2}}{=} -\frac{1}{2}$$

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b) E_1) $$l \overset{0^0}{=} e^{\lim\limits_{x\to\infty}\frac{1}{\ln x}\cdot\ln\left(\frac{\pi}{2}-\operatorname{arctg} x\right)}$$

E_2) $$l_1=\lim_{x\to\infty}\frac{\ln\left(\frac{\pi}{2}-\operatorname{arctg} x\right)}{\ln x} \overset{\frac{-\infty}{\infty}}{\underset{l'H}{=}}$$

$$=\lim_{x\to\infty}\frac{\frac{1}{\frac{\pi}{2}-\operatorname{arctg} x}\cdot\left(0-\frac{1}{1+x^2}\right)}{\frac{1}{x}}=\lim_{x\to\infty}-\frac{x}{\left(\frac{\pi}{2}-\operatorname{arctg} x\right)\cdot(1+x^2)}=$$

$$\overset{\frac{\infty}{0\cdot\infty}}{=} \lim_{x\to\infty}-\frac{\frac{x}{1+x^2}}{\frac{\pi}{2}-\operatorname{arctg} x} \overset{\frac{0}{0}}{\underset{l'H}{=}} \lim_{x\to\infty}-\frac{\frac{1(1+x^2)-x\cdot 2x}{(1+x^2)^2}}{0-\frac{1}{1+x^2}}$$

$$=\lim_{x\to\infty}\frac{\frac{1-x^2}{(1+x^2)^{\cancel{2}}}}{\frac{1}{\cancel{1+x^2}}}=\lim_{x\to\infty}\frac{1-x^2}{1+x^2}=-1$$

$$\Rightarrow l=e^{-1}=\frac{1}{e}$$