

FORMA RESTRÂNSĂ A UNEI LEGI

DE COMPOZITIE

1) $x \circ y = 2xy - 6x - 6y + 21$, n
cure $1020 \dots 2009$

2) $x \circ y = xy + 4x + 4y + 12$, n
cure $(-2013) \circ (-2012) \circ \dots \circ 2013$

3) $x * y = 5xy + 6x + 6y + 6$, rezolvati
 $\underbrace{x * x * \dots * x}_{2008 \text{ ori}} = -1$

4) $x \circ y = -xy + 2x + 2y - 2$, n
cure: $\frac{2012}{1} \circ \frac{2012}{2} \circ \dots \circ \frac{2012}{2012}$

5) $x * y = xy + 5x + 5y + 20$, n
cure $x * x * \dots * x$
 $\underbrace{\hspace{10em}}_{2016 \text{ ori}}$

6) $x * y = xy - 3x - 3y + 12$, n cure
 $\underbrace{x * x * \dots * x}_{2010 \text{ ori}}$, n $1 * 2x \dots * 2016$

7) $x * y = xy + x + y$, n cure
 $\underbrace{x * x * \dots * x}_{2016}$, n $1 * 2x \dots * 2016$

8) $x * y = xy - 5x - 5y + 30$, n
cure $\underbrace{x * x * \dots * x}_{2010}$, n $1 * 2x \dots * 2016$

9) $x * y = xy - x - y + 2$, n
cure $\underbrace{x * x * \dots * x}_{n \text{ ori}}$, n $(-100) \circ (-99) \circ \dots \circ 1000$

10) $x * y = xy + 2x + 2y + 2$, n cure
 $\underbrace{x * x * \dots * x}_{n \text{ ori}}$, n $(-100) \circ (-99) \circ \dots \circ 100$

$$\text{Fie } x \circ y = 2xy - 6x - 6y + 21, \forall x, y \in \mathbb{R}$$

$$\text{Calculați: } 1 \circ \sqrt{2} \circ \sqrt{3} \circ \dots \circ \sqrt{2009}$$

E₁) dacă înlocuim în relația inițială obținem
complicat

E₂) forma restată:

$$x \circ y = 2x(y-3) - 6(y-3) + 3$$

$$= (y-3)(2x-6) + 3$$

$$= (y-3) \cdot 2(x-3) + 3$$

$$x \circ y = 2(x-3)(y-3) + 3$$

E₃) dacă lui x valori a. i. se obținem $x \circ y$
constant

$$E_4) x=3 \Rightarrow 3 \circ y = 2 \cdot 0 + 3 = 3$$

$$y=3 \Rightarrow x \circ 3 = 2 \cdot 0 + 3 = 3$$

$$E_5) 1 \circ \sqrt{2} \circ \dots \circ \sqrt{2009} =$$

$$= \underbrace{(1 \circ \sqrt{2} \circ \dots \circ \sqrt{8})}_x \circ 3 \circ \underbrace{(\sqrt{10} \circ \dots \circ \sqrt{2009})}_y$$

$$= \underbrace{x \circ 3}_y \circ y = 3 \circ y = 3$$

$$E_6) N = 3$$

Pe \mathbb{R} avem legea $x \circ y = x(y+4) + 4y + 12$
Calculați $(-2013) \circ (-2012) \circ \dots \circ 2013$

E₁) dacă înlocuim direct nu iese nimic \rightarrow folosim
forma restrânsă

$$E_2) \quad x \circ y = x(y+4) + 4(y+4) - 4 \\ = (y+4)(x+4) - 4$$

$$E_3) \quad \text{Calculăm } x \circ (-4) = (x+4)(\underbrace{-4+4}_0) - 4 = -4 \\ -4 \circ x = (\underbrace{-4+4}_0)(x+4) - 4 = -4$$

$$E_4) \quad N = \underbrace{(-2013) \circ (-2012) \circ \dots \circ (-5) \circ (-4)}_x \circ \underbrace{(-3) \circ \dots \circ 2013}_y$$

$$N = x \circ (-4) \circ y$$

$$N = -4 \circ y = -4$$

Fie $x * y = 5xy + 6x + 6y + 6$, rezolvati in \mathbb{R}

$$\underbrace{x * x * \dots * x}_{2008 \text{ ori}} = -1$$

E₁) folosim formula restransă

$$\begin{aligned} x * y &= 5\left(x\left(y + \frac{6}{5}\right) + \frac{6}{5}y + \frac{6}{5}\right) = 5\left(x\left(y + \frac{6}{5}\right) + \frac{6}{5}\left(y + \frac{6}{5}\right) - \frac{36}{25} + \frac{6}{5}\right) = \\ &= 5\left(\left(y + \frac{6}{5}\right)\left(x + \frac{6}{5}\right) - \frac{6}{25}\right) = 5\left(x + \frac{6}{5}\right)\left(y + \frac{6}{5}\right) - \frac{6}{5} \end{aligned}$$

$$\Rightarrow \boxed{x * y = 5\left(x + \frac{6}{5}\right)\left(y + \frac{6}{5}\right) - \frac{6}{5}} \quad (1)$$

$$E_2) x * x = 5\left(x + \frac{6}{5}\right)\left(x + \frac{6}{5}\right) - \frac{6}{5} \Rightarrow x * x = 5\left(x + \frac{6}{5}\right)^2 - \frac{6}{5}$$

$$x * x * x = \left(5\left(x + \frac{6}{5}\right)^2 - \frac{6}{5}\right) * x = 5\left(\left(5\left(x + \frac{6}{5}\right)^2 - \frac{6}{5} + \frac{6}{5}\right)\left(x + \frac{6}{5}\right) - \frac{6}{5}\right)$$

$$\Rightarrow x * x * x = 5^2\left(x + \frac{6}{5}\right)^3 - \frac{6}{5}, \text{ arătam că}$$

$$\boxed{\underbrace{x * x * \dots * x}_{n \text{ ori}} = 5^{n-1}\left(x + \frac{6}{5}\right)^n - \frac{6}{5}} \text{ prin inducție}$$

$$E_3) \text{ I } P(2): x * x = 5^{2-1}\left(x + \frac{6}{5}\right)^2 - \frac{6}{5} \text{ aderă din (1)}$$

II Ip. $P(k)$ aderă $\xrightarrow{(?)} P(k+1)$ aderă unde

$$P(k): \underbrace{x * x * \dots * x}_{k \text{ ori}} = 5^{k-1}\left(x + \frac{6}{5}\right)^k - \frac{6}{5}$$

$$P(k+1): \underbrace{x * x * \dots * x}_{k+1 \text{ ori}} = 5^k\left(x + \frac{6}{5}\right)^{k+1} - \frac{6}{5}$$

$$\text{Sfârșit } P(k) \text{ aderă} \Rightarrow \underbrace{x * x * \dots * x}_{k \text{ ori}} = 5^{k-1}\left(x + \frac{6}{5}\right)^k - \frac{6}{5} \quad | * x \Rightarrow$$

$$\underbrace{x * x * \dots * x}_{k+1} = \left(5^{k-1}\left(x + \frac{6}{5}\right)^k - \frac{6}{5}\right) * x = \frac{1}{5} \left(5^k\left(x + \frac{6}{5}\right)^{k+1} - \frac{6}{5} - \frac{6}{5}\right) = 5^{k-1}\left(x + \frac{6}{5}\right)^{k+1} - \frac{6}{5}$$

$$= 5^k\left(x + \frac{6}{5}\right)^{k+1} - \frac{6}{5} \Rightarrow P(k+1) \text{ aderă} \Rightarrow P(n) \text{ aderă}, \forall n \geq 2$$

$$E_4) \text{ Se este: } 5^{2007}\left(x + \frac{6}{5}\right)^{2008} - \frac{6}{5} = -1 \Rightarrow 5^{2007}\left(x + \frac{6}{5}\right)^{2008} = \frac{1}{5} \Rightarrow$$

$$\Rightarrow \left(x + \frac{6}{5}\right)^{2008} = \left(\frac{1}{5}\right)^{2008} \Rightarrow x + \frac{6}{5} = \pm \frac{1}{5} < x = -\frac{1}{5} - \frac{6}{5} = -\frac{7}{5} \Rightarrow \left(-\frac{7}{5}\right)^{2008} = \left(\frac{1}{5}\right)^{2008}$$

$$\text{Fie } \mathcal{X} \circ y = -xy + 2x + 2y - 2$$

$$\text{Calculati } \frac{2012}{1} \circ \frac{2012}{2} \circ \dots \circ \frac{2012}{2012}$$

E₁) De obicei, când se cere o compunere cu mai mulți termeni, folosim forma restrânsă

$$E_2) \mathcal{X} \circ y = -x(y-2) + 2(y-2) + 2$$

$$\mathcal{X} \circ y = (y-2)(-x+2) + 2$$

$$x \circ y = (y-2) \cdot (-1)(x-2) + 2 \Rightarrow \mathcal{X} \circ y = -(x-2)(y-2) + 2$$

$$E_3) \text{ Observăm că } \mathcal{X} \circ 2 = -0 + 2 \Rightarrow \mathcal{X} \circ 2 = 2$$

$$2 \circ x = -0 + 2 \Rightarrow 2 \circ x = 2$$

$$E_4) N = \frac{2012}{1} \circ \frac{2012}{2} \circ \dots \circ \frac{2012}{2012} =$$

$$= \frac{2012}{1} \circ \frac{2012}{2} \circ \dots \circ \underbrace{\frac{2012}{1006}}_{=2} \circ \dots \circ \frac{2012}{2012}$$

$$= \underbrace{\left(\frac{2012}{1} \circ \frac{2012}{2} \circ \dots \circ \frac{2012}{1005} \right)}_{\mathcal{X}} \circ 2 \circ \underbrace{\left(\frac{2012}{1007} \circ \frac{2012}{1008} \circ \dots \circ \frac{2012}{2012} \right)}_{\mathcal{Y}}$$

$$E_5) N = (\mathcal{X} \circ 2) \circ \mathcal{Y} \xrightarrow{E_3)} 2 \circ \mathcal{Y} \xrightarrow{E_3)} 2$$