CALCULAREA LIMITELOR ȘIRURILOR DE INTEGRALE DEFINITE

Calculați $\lim_{n\to\infty} I_n$ în fiecare din situațiile următoare:

1) $I_n=\int_0^1 x^n \sin^2\frac{1}{x^2+1}\,dx$

2) $I_n=\int_0^1 \frac{x^{2n}}{1+x}\,dx$

3) $I_n=\int_0^1 \frac{x^n}{1+x^{2n}}\,dx$

4) $I_n=\int_0^n \frac{1}{2-\sin n}\,dx$

5) $I_n=\int_0^2 e^{x^2}\,dx$

6) $I_n=\int_0^1 \frac{\ln(x^n+1)}{x+1}\,dx$

7) $I_n=\int_1^2 \frac{1}{1+x^n}\,dx$

8) $I_n=\int_1^2 \frac{x^n}{1+x^n}\,dx$

9) $I_n=\int_0^1 \frac{n x^n}{x^n+1}\,dx$

10) $I_n=\int_0^{\pi/4} tg^{2n} x\,dx$

11) $I_n=\int_0^{\pi/3} \sin^n x\,dx$

12) $I_n=\int_n^{n+1} \frac{dx}{\sqrt{1+x^2}}\,dx$

13) $I_n=\int_0^1 \ln(1+x^n)\,dx$

14) $I_n=\int_0^1 x^n(1-x)^n\,dx$

15) $I_n=\int_1^e \frac{\ln x}{x^n}\,dx$

Calculați: $\lim_{n\to\infty} \int_0^1 \frac{x^n}{1+x^{2n}}\,dx$

$E_1)$ $x\in[0;1] \Rightarrow 0 \le \frac{x^n}{1+x^{2n}} \le x^n \quad \Big/ \int_0^1 \ldots\, dx$

$$\Rightarrow 0 \le \int_0^1 \frac{x^n}{1+x^{2n}}\,dx \le \int_0^1 x^n\,dx$$

$$\Rightarrow 0 \le I_n \le \frac{x^{n+1}}{n+1}\Big|_0^1 \Rightarrow$$

$$\Rightarrow 0 \le I_n \le \frac{1}{n+1}$$

$E_2)$ din criteriul cleștelui $\Rightarrow \lim_{n\to\infty} I_n = 0$

Calculați $\lim_{n\to\infty} I_n$ dacă

$$I_n=\int_0^1 x^n \cdot \sin^2 \frac{1}{x^2+1}\, dx$$

$E_1)$ din $I_n=\int_0^1 x^n \sin^2 \frac{1}{x^2+1}\, dx \Rightarrow x\in[0;1]$

$E_2)$ deoarece $\sin t \in[-1;1], \forall t\in\mathbb{R} \Rightarrow$

$\sin^2 t\in[0;1], \forall t\in\mathbb{R}$

$\Rightarrow \sin^2\frac{1}{x^2+1}\in[0;1], \forall x\in\mathbb{R}$

$E_3)$ $0\le \sin^2\frac{1}{x^2+1}\le 1 \;\Big/\cdot x^n>0$

$\Rightarrow 0\le x^n \sin^2\frac{1}{x^2+1}\le x^n \Big/ \int_0^1 \ldots dx$

$\Rightarrow 0\le I_n \le \frac{x^{n+1}}{n+1}\Big|_0^1$

$\Rightarrow 0\le I_n\le \frac{1}{n+1}$

$\searrow 0 \qquad \swarrow 0$

$\Rightarrow \lim_{n} I_n = 0$

Calculați $\lim\limits_{n\to\infty} I_n$ dacă

$$I_n=\int_1^2 \frac{x^n}{1+x^n}\,dx$$

E1) $I_n=\int_1^2 \frac{x^n}{1+x^n}\,dx \Rightarrow x\in[1;2]$

E2) $1\le x\le 2 \;|\,()^n \Rightarrow 1\le x^n\le 2^n \;|+1$

$\Rightarrow 2\le 1+x^n\le 2^n+1$

$\Rightarrow \frac{1}{2^n+1}\le \frac{1}{1+x^n}\le\frac{1}{2} \;\Big|\cdot x^n$

$\Rightarrow \frac{x^n}{2^n+1}\le \frac{x^n}{1+x^n}\le \frac{x^n}{2} \;\Big|\int_1^2 \dots dx$

$\Rightarrow \frac{1}{2^n+1}\,\frac{x^{n+1}}{n+1}\Big|_1^2 \le I_n\le \frac{1}{2}\,\frac{x^{n+1}}{n+1}\Big|_1^2$

$\Rightarrow \frac{1}{2^n+1}\left(\frac{2^{n+1}}{n+1}-\frac{1}{n+1}\right)\le I_n\le \frac{1}{2}\left(\frac{2^{n+1}}{n+1}-\frac{1}{n+1}\right)$

$\Leftrightarrow \underbrace{\frac{2^n\cdot 2}{2^n+1}}_{2}\cdot\underbrace{\frac{1}{n+1}}_{0}-\underbrace{\frac{1}{2^n+1}\cdot\frac{1}{n+1}}_{0}\le I_n\le \frac{1}{2}\left(\underbrace{\frac{2^{n+1}}{n+1}}_{\infty}-\frac{1}{n+1}\right)$

$\Rightarrow 0\le \lim\limits_n I_n\le \infty \Rightarrow$ nu putem găsi $\lim\limits_n I_n$

E3) $I_n=\int_1^2 \frac{x^n+1-1}{1+x^n}\,dx=\int_1^2 1\,dx-\int_1^2\frac{1}{1+x^n}\,dx=1-J_n$

E4) Af $\lim\limits_n J_n$, $\quad 0\le\frac{1}{1+x^n}\le\frac{1}{x^n}\;\Big|\int_1^2\dots dx$

$\Rightarrow 0\le J_n\le \frac{x^{-n+1}}{-n+1}\Big|_1^2 \Rightarrow 0\le J_n\le \overset{0}{\frac{1}{-n+1}}\,\overset{0}{\left(2^{-n+1}-1\right)}$

$\Rightarrow \lim\limits_n J_n=0\Rightarrow \lim\limits_n I_n=1-0=1$

Calculați $\lim_{n\to\infty} I_n$ dacă

$$I_n = \int_1^2 \frac{1}{1+x^n}\,dx$$

$E_1)$ $I_n = \int_1^2 \frac{1}{1+x^n}\,dx \Rightarrow x \in [1;2]$

$E_2)$ $1 \le x \le 2 \,/\!/ (\ )^n \Rightarrow 1 \le x^n \le 2^n \,/ +1 \Rightarrow 2 \le x^n+1 \le 2^n+1$

$$\Rightarrow \frac{1}{2^n+1} \le \frac{1}{x^n+1} \le \frac{1}{2} \;/\!/ \int_1^2 \dots dx$$

$$\Rightarrow \frac{1}{2^n+1} \cdot x\Big|_1^2 \le I_n \le \frac{1}{2} \cdot x\Big|_1^2$$

$$\Rightarrow \underbrace{\frac{1}{2^n+1}(2-1)}_{\to 0} \le I_n \le \underbrace{\frac{1}{2}(2-1)}_{\to 1}$$

$\Rightarrow \lim_n I_n$ nu o putem găsi, căutăm altă încadrare

$E_3)$ $x \in [1;2] \Rightarrow 0 \le \frac{1}{1+x^n} \le \frac{1}{x^n} \;/\!/ \int_1^2 \dots dx$

$$\Rightarrow 0 \le I_n \le \int_1^2 x^{-n}\,dx \Rightarrow$$

$$0 \le I_n \le \frac{x^{-n+1}}{-n+1}\Big|_1^2 \Rightarrow 0 \le I_n \le \underbrace{\overbrace{\frac{1}{-n+1}}^{\to 0}\big(\overbrace{2^{-n+1}}^{\to 0} - 1\big)}_{\to 0}$$

$\to 0$

$$\Rightarrow \lim_n I_n = 0$$