RELAȚIE DE RECURENȚĂ PENTRU

$$I_n = \int_\alpha^\beta (ax^2+bx+c)^n dx$$

Determinați relația de recurență pentru I_n în fiecare din situațiile următoare:

1) $I_n = \int_3^4 (x^2-7x+12)^n dx$

2) $I_n = \int_1^2 (x^2-3x+2)^n dx$

3) $I_n = \int_{-1}^1 (x^2-1)^n dx$

4) $I_n = \int_0^2 (x^2-2x)^n dx$

5) $I_n = \int_{-1}^{-2} (x^2+3x+2)^n dx$

6) $I_n = \int_1^3 (x^2-4x+3)^n dx$

7) $I_n = \int_1^4 (x^2-5x+4)^n dx$

8) $I_n = \int_{-1}^2 (x^2-x-2)^n dx$

9) $I_n = \int_{1/2}^1 (2x^2-3x+1)^n dx$

10) $I_n = \int_{1/2}^{3/2} (4x^2-8x+3)^n dx$

Fie $I_n = \int_3^4 (x^2-7x+12)^n dx$, găsiți o relație de recurență pt I_n

E1) $I_n = \int_3^4 (x^2-7x+12)^n dx$, $f=(x^2-7x+12$

$$f=(x^2-7x+12)^n \Rightarrow f' = n(x^2-7x+12)^{n-1}\cdot(2x-7)$$

$$g' = 1 \Rightarrow g = x$$

E2) $I_n = x(x^2-7x+12)^n\Big|_3^4 - n\int_3^4 (x^2-7x+12)^{n-1}(2x^2-7x)dx$

$$I_n = 0-0-n\cdot 2\int_3^4 (x^2-7x+12)^{n-1}\left(x^2-\frac{7}{2}x\right)dx$$

$$I_n = -2n\int_3^4 (x^2-7x+12)^{n-1}\left(\underbrace{x^2-7x+12}+\underbrace{\overset{3}{7x}-12-\frac{7}{2}x}\right)dx$$

$$I_n = -2n\left(\int_3^4 (x^2-7x+12)^n dx + \int_3^4 \left(\frac{7x}{2}-12\right)(x^2-7x+12)^{n-1}dx\right)$$

$$I_n = -2n\left(I_n + \frac{7}{2}\int_3^4 (x^2-7x+12)^{n-1}\left(x-\frac{12}{\frac{7}{2}}\right)dx\right)$$

E3) $I_n = -2nI_n - 7n\int_3^4 (x^2-7x+12)^{n-1}\left(x-\frac{24}{7}\right)dx$

$$I_n(1+2n) = -\frac{7n}{2}\int_3^4 (x^2-7x+12)^{n-1}\cdot\left(2x-\frac{48}{7}\right)dx$$

$$I_n(2n+1) = -\frac{7n}{2}\int_3^4 (x^2-7x+12)^{n-1}\left(2x-7+7-\frac{48}{7}\right)dx$$

$$I_n(2n+1) = -\frac{7n}{2}\left(\int_3^4 (x^2-7x+12)^{n-1}\cdot(2x-7)dx + \int_3^4 (x^2-7x+12)^{n-1}\cdot\frac{1}{7}dx\right)$$

$$I_n(2n+1) = -\frac{7n}{2}\left(\int_3^4 (x^2-7x+12)^{n-1}(x^2-7x+12)'dx + \frac{1}{7}I_{n-1}\right)$$

$$I_n(2n+1) = -\frac{7n}{2}\left(\frac{(x^2-7x+12)^n}{n}\Big|_3^4 + \frac{1}{7}I_{n-1}\right)$$

$$I_n(2n+1) = -\frac{7n}{2}(0-0) - \frac{n}{2}I_{n-1}\Big|\cdot 2 \Rightarrow \boxed{I_n(4n+2) = -nI_{n-1}}$$

s-a folosit $\int f^{n-1}f' = \frac{f^n}{n}$

15 (F 5)

Fie $f(x) = x^2 - 3x + 2$

Se cere: $(4n+2)\int_1^2 f^n(x)\,dx + n\int_1^2 f^{n-1}(x)\,dx = 0$

$E_1)\ I_n = \int_1^2 (x^2-3x+2)^n\,dx$, se cere $(4n+2)I_n + n\,I_{n-1} = 0$

$f(x) = (x^2-3x+2)^n \Rightarrow f'(x) = n(x^2-3x+2)^{n-1}\cdot(2x-3)$

$g'(x) = 1 \Rightarrow g(x) = x$

$E_2)\ I_n = x(x^2-3x+2)^n\Big|_1^2 - n\int_1^2 (x^2-3x+2)^{n-1}(2x^2-3x)\,dx$

$I_n = 0 - 0 - n\cdot 2\int_1^2 (x^2-3x+2)^{n-1}\left(x^2 - \frac{3}{2}x\right)dx$

$I_n = -2n\int_1^2 (x^2-3x+2)^{n-1}\left(x^2-3x+2 + \overset{2}{\cancel{3}}x - 2 - \frac{3}{2}x\right)$

$I_n = -2n\left(\int_1^2 (x^2-3x+2)^n\,dx + \int_1^2 (x^2-3x+2)^{n-1}\left(\frac{3x}{2} - 2\right)dx\right)$

$E_3)\ I_n = -2n\left(I_n + \frac{3}{2}\int_1^2 (x^2-3x+2)^{n-1}\left(x - \frac{4}{3}\right)dx\right)$

$I_n = -2nI_n - \cancel{2}n\cdot\frac{3}{\cancel{2}}\cdot\frac{1}{2}\int_1^2 (x^2-3x+2)^{n-1}\left(2x-3 - \frac{8}{3} + 3\right)dx$

$I_n(2n+1) = -\frac{3n}{2}\left(\int_1^2 (x^2-3x+2)^{n-1}(x^2-3x+2)'\,dx + \int_1^2 (x^2-3x+2)^{n-1}\frac{1}{3}\,d\ldots\right.$

$E_4)$ Dar, $\int f^{n-1}\cdot f' = \frac{f^n}{n}$

$\Rightarrow I_n(2n+1) = -\frac{3n}{2}\left(\frac{(x^2-3x+2)^n}{n}\Big|_1^2 + \frac{1}{3}I_{n-1}\right)$

$I_n(2n+1) = -\frac{3n}{2}\left(0 - 0 + \frac{1}{3}I_{n-1}\right) \Rightarrow I_n(2n+1) = -\frac{n}{2}I_{n-1}\ \Big|\cdot 2$

$E_5)\ 2I_n(2n+1) = -n\,I_{n-1} \Rightarrow I_n(4n+2) + n\,I_{n-1} = 0$